Restricted Turing Machines and Language Recognition

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Hennie Machines & C

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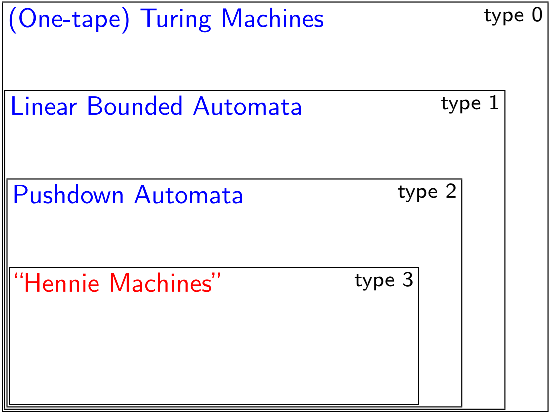
Limited Automata & C

The Chomsky Hierarchy



Part II: One-Tape TMs with Rewriting Restrictions

Outline

- Limited automata
- Equivalence with CFLs
- Determinism vs nondeterminism
- Descriptional complexity aspects
- 1-limited automata and regular languages
- Related models

Limited Automata [Hibbard '67]

One-tape Turing machines with restricted rewritings

Definition

Fixed an integer $d \geq 1$, a *d-limited automaton* is

- a one-tape Turing machine
- which is allowed to rewrite the content of each tape cell *only in the first d visits*

Computational power

- For each $d \geq 2$, d-limited automata characterize context-free languages [Hibbard '67]
- 1-limited automata characterize regular languages [Wagner&Wechsung '86]

Example: Balanced Parentheses

$\triangleright$	(	)	(	(	(	)	)	)	$\triangleleft$

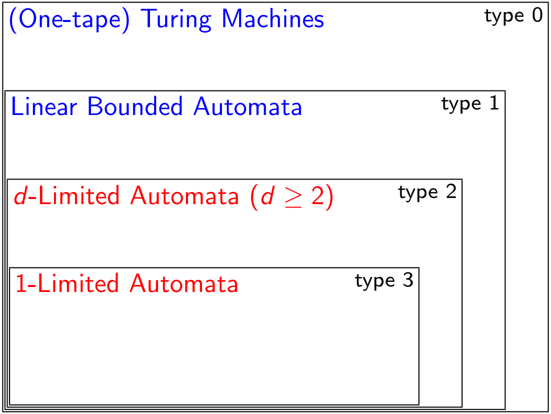
(i) Move to the right to search a closed parenthesis
(ii) Rewrite it by x
(iii) Move to the left to search an open parenthesis
(iv) Rewrite it by x
(v) Repeat from the beginning

Special cases:

(i') If in (i) the right end of the tape is reached then scan all the tape and *accept* iff all tape cells contain x
(iii') If in (iii) the left end of the tape is reached then *reject*

Each cell is rewritten only in the first 2 visits!

The Chomsky Hierarchy



Why Each CFL is Accepted by a 2-LA [P.&Pisoni '14]

Main tool:

Theorem ([Chomsky&Schützenberger '63])

Every context-free language $L \subseteq \Sigma^*$ can be expressed as

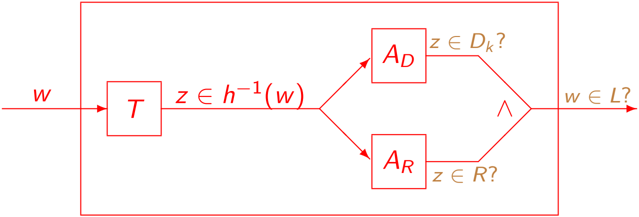
$$L = h(D_k \cap R)$$

where, for $\Omega_k = \{(_1,)_1, (_2,)_2, \ldots, (_k,)_k\}$:

- $D_k \subseteq \Omega_k^*$ *is a Dyck language*
- $R \subseteq \Omega_k^*$ *is a regular language*
- $h : \Omega_k \to \Sigma^*$ *is an homomorphism*

Furthermore, it is possible to restrict to *non-erasing* homomorphisms [Okhotin '12]

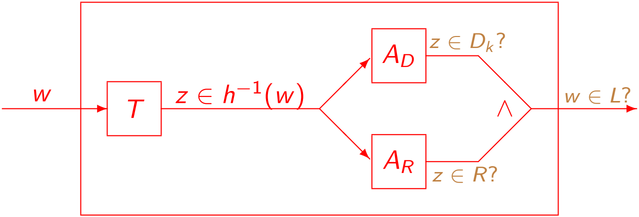
Why Each CFL is Accepted by a 2-LA



L context-free language, with $L = h(D_k \cap R)$

- T nondeterministic transducer computing h^{-1}
- A_D 2-LA accepting the Dyck language D_k
- A_R finite automaton accepting R

Why Each CFL is Accepted by a 2-LA



$$\underbrace{\boxed{\;u_1\;|\;u_2\;|\;\cdots\;|\;u_k\;}}_{\text{input of } T}$$

$z = \sigma_1\sigma_2\cdots\sigma_k \in h^{-1}(w)$

$h(\sigma_i) = u_i$

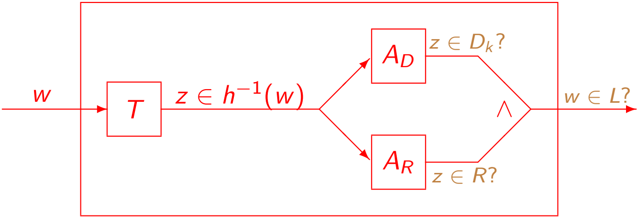
$$\underbrace{\boxed{\;\#\#\#\#\sigma_1\;|\;\#\#\sigma_2\;|\;\cdots\;|\;\#\#\#\sigma_k\;}}_{\text{(padded) input of } A_D \text{ and } A_R}$$

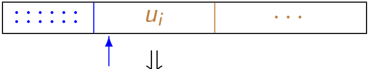
Non erasing homomorphism!

Not stored into the tape!

Each σ_i is produced “on the fly”

Why Each CFL is Accepted by a 2-LA





$$w = \cdots u_i \cdots$$

$$\Downarrow$$

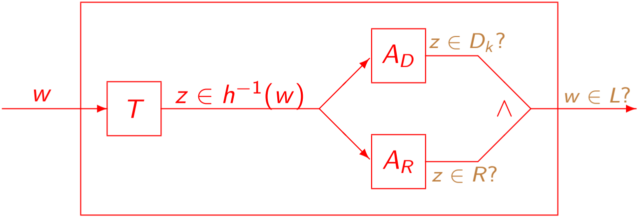
$$\boxed{\#\#\#\#\sigma_i} \qquad h(\sigma_i) = u_i$$

$$\Downarrow$$

$$\boxed{\#\#\#\#\gamma_i} \qquad \gamma_i\text{: first rewriting by } A_D$$

- On the tape, u_i is replaced directly by $\#\#\#\#\gamma_i$
- One move of A_R on input σ_i is also simulated

Why Each CFL is Accepted by a 2-LA



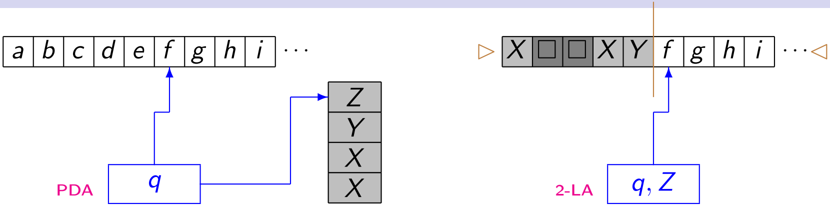
The resulting machine is a 2-LA recognizing the given CFL

Problems:

- What about the size of the resulting machine?
- What about the case of *deterministic* CFLs?

PDAs vs Limited Automata

Simulation of Pushdown Automata by 2-Limited Automata



Normal form for (D)PDAs:

- at each step, the stack height increases at most by 1
- ϵ-moves cannot push on the stack

Each PDA can be simulated by an equivalent 2-LA

- Polynomial size
- Determinism is preserved

Simulation of 2-Limited Automata by Pushdown Automata

Problem

What about the converse simulation, namely that of 2-LAs by PDAs?

[Hibbard '67]

Original simulation

[P.&Pisoni '15]

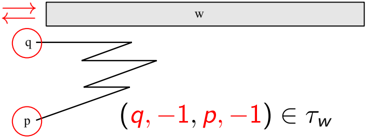
Reformulation

- Exponential cost
- Determinism is preserved (extra costs)

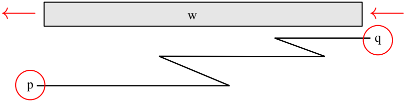
Transition Tables of 2-LAs

- Fixed a 2-limited automaton
- *Transition table* τ_w — w is a "frozen" string

$$\tau_w \subseteq Q \times \{-1,+1\} \times Q \times \{-1,+1\}$$



$(q, -1, p, -1) \in \tau_w$



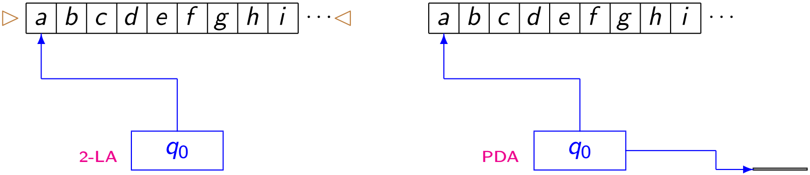
$(q, +1, p, -1) \in \tau_w$

$(q, d', p, d'') \in \tau_w$ iff M on a tape segment containing w has a computation path:

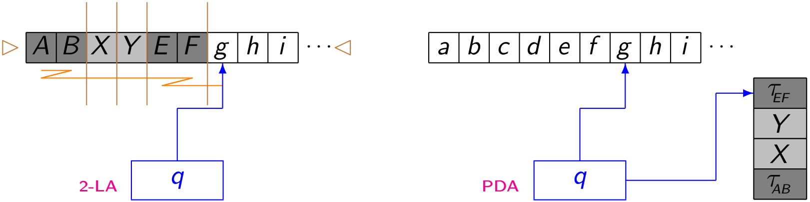
- entering the segment in q from d'
- exiting the segment in p to d''
- left $= -1$, right $= +1$

Simulation of 2-LAs by PDAs

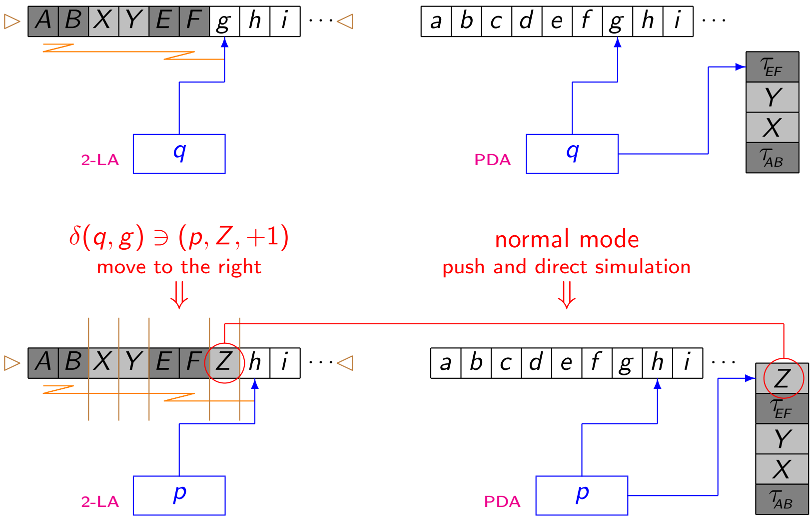
Initial configuration



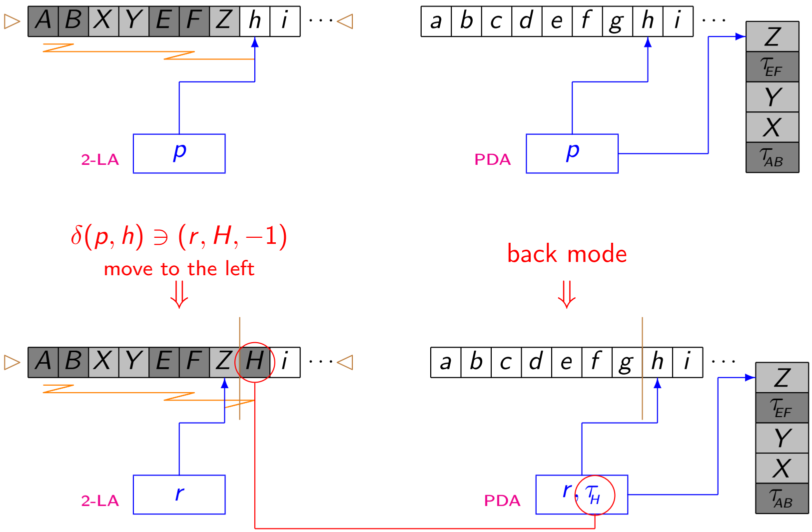
After some steps...



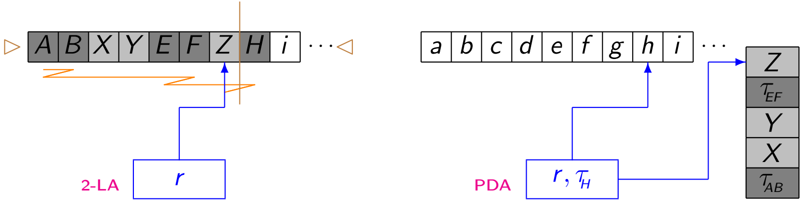
Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



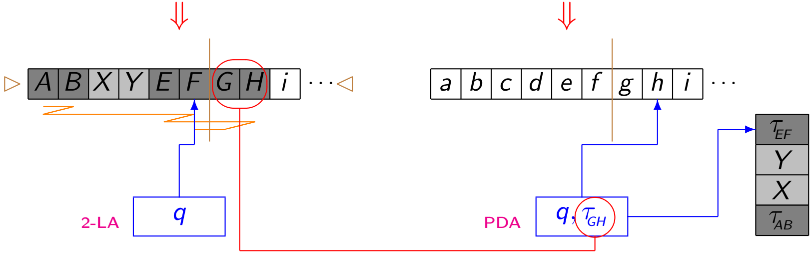
Simulation of 2-LAs by PDAs



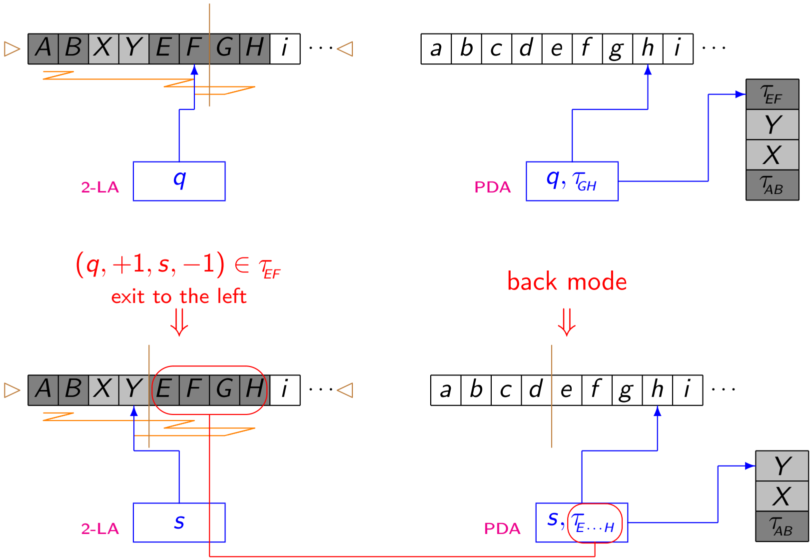
$\delta(r, Z) \ni (q, G, -1)$

move to the left

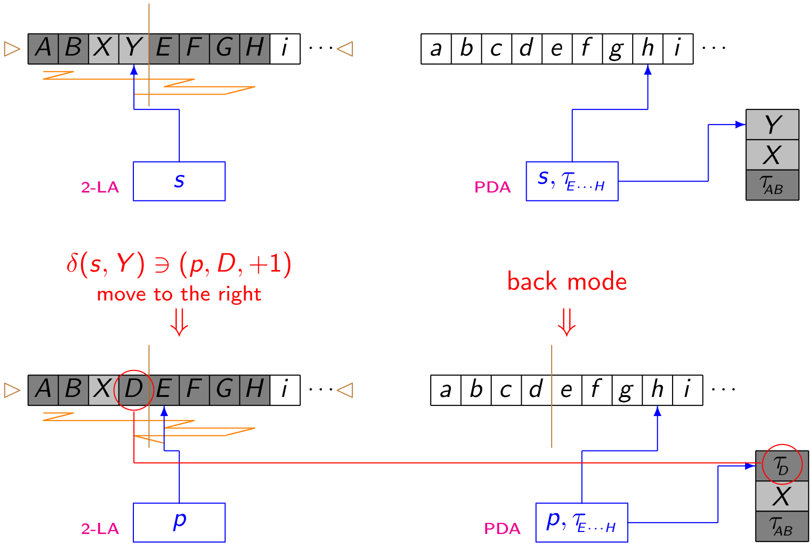
back mode



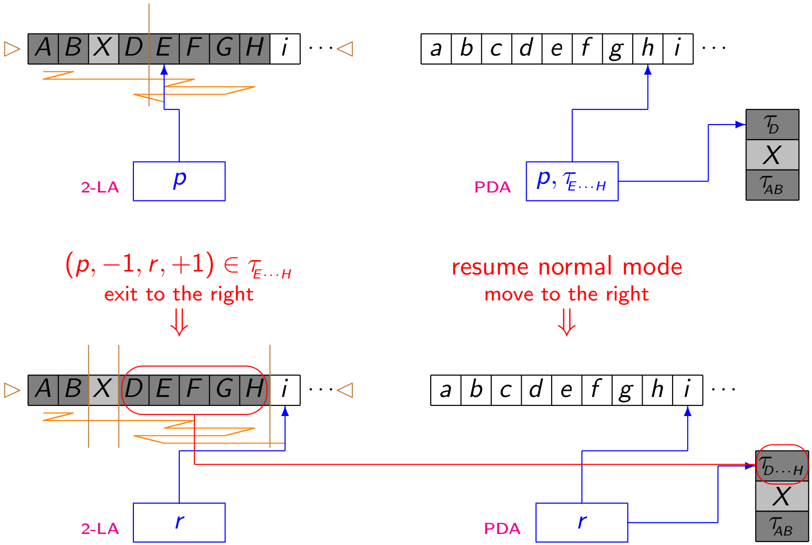
Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs



Simulation of 2-LAs by PDAs

Summing up...

Given a 2-LA M with:

- n states
- m symbol working alphabet

At most 2^{4n^2} many different tables!

Resulting PDA:

- States
  Normal mode: states of M
  Back mode: (q, τ)
  q state of M, τ transition table
- Pushdown symbols
  - Tape symbols of M
  - Transition tables
- Each move can increase the stack height at most by 1

States
$2n(2^{4n^2} + 1) + 1$

Pushdown symbols
$m + 2^{4n^2}$

2-LAs $\rightarrow$ PDAs
Exponential cost

Optimality: the Witness Languages K_n

Given $n \geq 1$:

$$\underbrace{a_1 \quad a_2 \quad \cdots \quad a_n}_{x_1} \quad \cdots \quad \underbrace{a_{n+1}\, a_{n+2} \quad \cdots \quad a_{2n}}_{x_k} \quad \underbrace{b_1 \quad b_2 \quad \cdots \quad b_n}_{x}$$

$\cdots$

At least n of these blocks are equal
to the last block x

$$\begin{aligned} K_n \quad = \{x_1 x_2 \cdots x_k x \mid \quad & k \geq 0,\ x_1, x_2, \ldots, x_k, x \in \{0,1\}^n, \\ & \exists i_1 < i_2 < \cdots < i_n \in \{1, \ldots, k\}, \\ & x_{i_1} = x_{i_2} = \cdots = x_{i_n} = x \,\} \end{aligned}$$

Example ($n = 3$): $\quad 0\ 0\ 1 | 1\ 1\ 0 | 0\ 1\ 1 | 1\ 1\ 0 | 1\ 1\ 0 | 1\ 1\ 1 | 1\ 1\ 0$

How to Recognize K_n

0 0 1 1 1 0 0 1 1 1 1 0 1 1 0 1 1 1 1 1 0 $\qquad (n = 3)$

1. Scan all the tape from left to right
2. Start to move to the left and mark the rightmost n symbols
3. Compare each block of length n (from the right), symbol by symbol, with the last block
4. When the left end of the tape is reached accept if and only if the number of block equal to the last one is $\geq n$

Complexity:

- K_n is accepted by a deterministic 2-LA with $O(n^2)$ states and a fixed working alphabet
- Each PDA accepting K_n has size at least exponential in n (Proof based on the *interchange lemma* for CFLs)

Simulation of 2-LAs by PDAs

Cost of the simulation

- Exponential size for the simulation of 2-LAs by PDAs
- Optimal

Computational Power of Limited Automata

From the simulations:

- 2-Limited Automata $\equiv$ CFLs

What about d-Limited Automata, with $d > 2$?

- They are still characterize CFLs [Hibbard '67]
- They can be simulated by exponentially larger PDAs [Kutrib&P.&Wendlandt subm.]

What about 1-Limited Automata?

- Regular languages [Wagner&Wechsung '86]

Determinism vs Nondeterminism

- Determinism is preserved by the exponential simulation of 2-limited automata by PDAs
  provided that the input of the PDA is right end-marked
- *Without end-marker:* double exponential simulation
- *Conjecture:* this cost cannot be reduced
- The converse simulation also preserve determinsm

Deterministic 2-Limited Automata $\equiv$ DCFLs

[P.&Pisoni '15]

Determinism vs Nondeterminism

What about *deterministic* d-Limited Automata, $d > 2$?

- $L = \{a^n b^n c \mid n \geq 0\} \cup \{a^n b^{2n} d \mid n \geq 0\}$
  is accepted by a *deterministic* 3-LA, but is not a DCFL
- Infinite hierarchy [Hibbard '67]

  For each $d \geq 2$ there is a language which is accepted by a deterministic d-limited automaton and that cannot be accepted by any deterministic $(d-1)$-limited automaton

1-Limited Automata

Simulation of 1-Limited Automata by Finite Automata

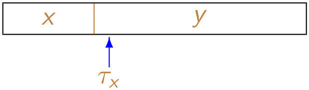
Main idea: transformation of *two-way* NFAs into *one-way* DFAs

[Shepherdson '59]

- First visit to a cell: direct simulation
- Further visits: *transition tables*

  for $x \in \Sigma^*$, $\tau_x \subseteq Q \times Q$: $(p, q) \in \tau_x$ iff 

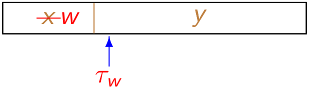
- Finite control of the DFA which simulates the two-way NFA:



  - transition table of the already scanned input prefix
  - set of possible current states

Simulation of 1-Limited Automata by Finite Automata

Simulation of 1-LAs: [Wagner&Wechsung '86]



- The transition table depends on the string used to rewrite the input prefix x
- This string was nondeterministically chosen by the 1-LA

The simulating DFA keeps in its finite control a *sets of transition tables*

1-Limited Automata $\to$ Finite Automata: Upper Bounds

Theorem

Let M be a 1-LA with n states.

- *There exists an equivalent DFA with $2^{n \cdot 2^{n^2}}$ states.*
- *There exists an equivalent NFA with $n \cdot 2^{n^2}$ states.*

If M is deterministic then there exists an equivalent DFA with no more than $n \cdot (n+1)^n$ states.

	DFA	NFA
nondet. 1-LA	$2^{n \cdot 2^{n^2}}$	$n \cdot 2^{n^2}$
det. 1-LA	$n \cdot (n+1)^n$	$n \cdot (n+1)^n$

These upper bounds do not depend on the alphabet size of M!

The gaps are optimal!

Optimality: the Witness Languages [P.&Pisoni '14]

Fixed $n \geq 1$:

$$\underbrace{a_1 \quad a_2 \quad \cdots \quad a_n}_{x_1} \; \underbrace{a_{n+1}\, a_{n+2} \quad \cdots \quad a_{2n}}_{x_2} \quad \cdots \quad \underbrace{a_{\dots} \quad a_{\dots} \quad \cdots \quad a_{kn}}_{x_k}$$

At least n of these blocks are equal

$$\begin{aligned} L_n \quad = \{x_1 x_2 \cdots x_k \mid \quad & k \geq 0,\ x_1, x_2, \ldots, x_k \in \{0,1\}^n, \\ & \exists i_1 < i_2 < \cdots < i_n \in \{1, \ldots, k\}, \\ & x_{i_1} = x_{i_2} = \cdots = x_{i_n} \} \end{aligned}$$

Example ($n = 3$): 0 0 1|1 1 0|0 1 1|1 1 0|1 1 0|1 1 1|0 1 1

How to Recognize L_n: 1-Limited Automata

$$0\,0\,1 | \hat{1}\,1\,0 | 0\,1\,1 | \hat{1}\,1\,0 | \hat{1}\,1\,0 | 1\,1\,1 | 0\,1\,1 \qquad (n = 3)$$

- Nondeterministic strategy:
  Guess the leftmost positions of n input blocks containing the same factor and *Verify*
- Implementation (3 tape scans):
  1. Mark n tape cells
  2. Count the tape modulo n to check whether or not:
     - the input length is a multiple of n, and
     - the marked cells correspond to the leftmost symbols of some blocks of length n
  3. Compare, symbol by symbol, each two consecutive blocks of length n that start from the marked positions
- $O(n)$ states

How to Recognize L_n: Deterministic Finite Automata

- Idea:
  - For each $x \in \{0,1\}^n$ count how many blocks coincide with x
  - Accept if and only if one of the counters reaches the value n
- State upper bound:
  - Finite control:
    a counter (up to n) for each possible block of length n
  - There are 2^n possible different blocks of length n
  - Number of states double exponential in n
    more precisely $(2^n - 1) \cdot n^{2^n} + n$
- State lower bound:
  - n^{2^n} (standard distinguishability arguments)

The state gap between 1-LAs and DFAs is double exponential!

How to Recognize L_n: Nondeterministic Finite Automata

- Idea:
  - *Guess* $x \in \{0, 1\}^n$
  - *Verify* whether or not n blocks in the input contains x
- State upper bound:
  - Finite control: a counter $\leq n$ for the occurrences of x, and a counter modulo n for input positions
  - Number of states: $O(n^2 \cdot 2^n)$
- State lower bound:
  - $n^2 \cdot 2^n$ (fooling set technique)

Nondetermism vs. Determinism in 1-LAs

$$\begin{tikzcd}
\begin{array}{r} L_n\text{: } O(n) \\ \text{states} \end{array} & \text{1-LA} \arrow[rr, "\exp\exp"] \arrow[d, "\exp"'] & & \text{DFA} & \begin{array}{r} L_n\text{: } \geq n^{2^n} \\ \text{states} \end{array} \\
\begin{array}{r} L_n\text{: } \geq \exp(n) \\ \text{states} \end{array} & \text{det-1-LA} \arrow[urr, "\exp"'] & & &
\end{tikzcd}$$

Corollary

Removing nondeterminism from 1-LAs requires exponentially many states

Cfr. Sakoda and Sipser question [Sakoda&Sipser '78]:

How much it costs in states to remove nondeterminism
from two-way finite automata?

Strongly Limited Automata

Different Restrictions

- Dyck languages are accepted without fully using capabilities of 2-limited automata
- Chomsky-Schützenberger Theorem: Recognition of CFLs can be reduced to recognition of Dyck languages

Question

Is it possible to restrict 2-limited automata without affecting their computational power?

YES!

Forgetting Automata
[Jancar&Mráz&Plátek '96]

- The content of any cell can be erased in the 1st or 2nd visit (using a fixed symbol)
- No other changes of the tape are allowed

Strongly Limited Automata [P.'15]

- Model inspired by the algorithm used by 2-limited automata to recognize Dyck languages
- Restrictions on
  - state changes
  - head reversals
  - rewriting operations

Dyck Language Recognition

▷	(	)	(	[	[	]	]	)	◁

- Moves to the right:
  - to search a closed bracket — Only one state q_0!
- Moves to the left:
  - to search an open bracket — One state for each type of bracket!
  - to check the tape content in the final scan from right to left
- Rewritings:
  - each closed bracket is rewritten in the first visit
  - each open bracket is rewritten in the second visit
  - no rewritings in the final scan

Strongly Limited Automata

- Alphabet
  - Σ input
  - Γ working

- States and moves
  - q_0 initial state, moving from left to right
    - $\dashrightarrow$ *move to the right*
    - ${}_q\xleftarrow{X}$ write $X \in \Gamma$, enter state $q \in Q_L$, *turn to the left*
  - Q_L moving from right to left
    - $\dashleftarrow$ *move to the left*
    - $\xleftarrow{X}$ write X, do not change state, *move to the left*
    - $\xrightarrow{X}_{q_0}$ write X, enters state q_0, *turn to the right*
  - Q_Υ final scan
    when $\lhd$ is reached move from right to left and
    test the membership of the tape content to a "local" language

Strongly Limited Automata: Palindromes

$\Sigma = \{a, b\}$, $\Gamma = \{X, Y, Z\}$
q_0
$Q_L = \{q_a, q_b\}$

$\triangleright$	a	b	b	b	a	$\triangleleft$

Transitions:

q_0 $\dashrightarrow$ *move to the right*

other possibility in cell not yet rewritten:

$_{q_\sigma}\xleftarrow{X}$ write $X \in \Gamma$, enter state $q_\sigma \in Q_L$, *turn to the left*

q_σ moving from right to left

cells already rewritten: $\dashleftarrow$ *move to the left*

cells containing $\gamma \in \{a, b\}$, nondeterministically select between:

$\xleftarrow{Z}$ write Z, do not change state, *move to the left*

$\xrightarrow{Y}_{q_0}$ write Y, enters state q_0, *turn to the right* (only if $\gamma = \sigma$)

Strongly Limited Automata: Palindromes

$\Sigma = \{a, b\}$, $\Gamma = \{\mathsf{X}, \mathsf{Y}, \mathsf{Z}\}$
q_0
$Q_L = \{q_a, q_b\}$

$\triangleright$ | Y | Y | Z | X | X | $\triangleleft$

Final phase:

- The string between the end-markers should belong to

$$\mathsf{Y}^*\mathsf{Z}\mathsf{X}^* + \mathsf{Y}^*\mathsf{X}^*$$

  with the exceptions of inputs of length ≤ 1

- The following two-letter factors are allowed:

  $\triangleright\mathsf{Y}$ $\quad$ YY $\quad$ YZ $\quad$ ZX $\quad$ YX $\quad$ XX $\quad$ $\mathsf{X}\triangleleft$

  $\triangleright a$ $\quad$ $\triangleright b$ $\quad$ $a\triangleleft$ $\quad$ $b\triangleleft$ $\quad$ $\triangleright\triangleleft$

Strongly Limited Automata

- Computational power: same as 2-limited automata (CFLs)
- Descriptional power: the sizes of equivalent
  - CFGs
  - PDAs
  - strongly limited automata

  are polynomially related

  - 2-limited automata can be exponentially smaller
- CFLs $\rightarrow$ strongly limited automata:

  conversion from CFGs which heavily uses nondeterminism

Determinism vs Nondeterminism

What is the power of *deterministic* strongly limited automata?

- Each deterministic strongly limited automaton can be simulated by a deterministic 2-LA
- Deterministic languages as

  $L_1 = \{ca^n b^n \mid n \geq 0\} \cup \{da^{2n} b^n \mid n \geq 0\}$
  $L_2 = \{a^n b^{2n} \mid n \geq 0\}$

  are not accepted by *deterministic strongly limited automata*

Proper subclass of deterministic context-free languages

Determinism vs Nondeterminism: a Small Change

- Moving to the right, a strongly limited automaton can use only q_0
- A possible modification:

  a set of states Q_R used while moving to the right

  - the simulation by PDAs remains polynomial
  - $L_1 = \{ca^n b^n \mid n \geq 0\} \cup \{da^{2n} b^n \mid n \geq 0\}$

    $L_2 = \{a^n b^{2n} \mid n \geq 0\}$

    are accepted by *deterministic devices*

Problem

What is the class of languages accepted
by the deterministic version of devices so obtained?

Final Remarks

Active Visits ad Return Complexity

Active visit of a tape cell: any visit changing the content

Return Complexity

Maximum number of visits to a tape cell counted starting from the *first* active visit [Wechsung '75]

- ret-c(1): regular languages
- ret-c(d), $d \geq 2$: context-free languages
- ret-c(2) *deterministic:* not comparable with DCFLs

Dual Return Complexity

Maximum number of visits to a tape cell counted up to the *last* active visit

dret-c(d) $\equiv$ d-limited automata

ret-c($f(n)$)=dret-c($f(n)$) =1AuxPDA($f(n)$)

[Wechsung&Brandstädt '79]

Thank you for your attention!