Two-Way Automata and Descriptional Complexity

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IFIP TC1, Working Group 1.2, *Descriptional Complexity*

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI INFORMATICA

What is Descriptional Complexity?

A Classical Example: NFAs vs DFAs

Formal language point of view:

- The class of languages recognized by NFAs coincides with the class of languages recognized by DFAs

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- Each n-state NFA can be simulated by a DFA *with* 2^n *states* [Rabin&Scott '59]
- For each integer n there exists a language L_n s.t.:
  - L_n is accepted by an n-state NFA
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Descriptional Complexity

Given

- $\mathcal{C}$ a class of languages
- $\mathcal{S}$ a formal system (e.g., class of devices, class of grammars,...) able to represent all the languages in $\mathcal{C}$

What is the *size* of the representations of the languages in $\mathcal{C}$ by the system $\mathcal{S}$?

Descriptional complexity compares different descriptions of a same class of languages:

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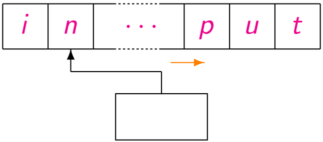
One-way and Two-way Automata

Finite State Automata



One-way version

At each step the input head is moved
one position to the right

- 1DFA: *deterministic* transitions
- 1NFA: *nondeterministic* transitions

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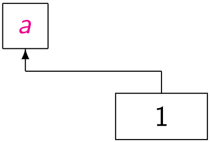

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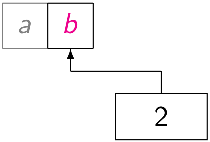

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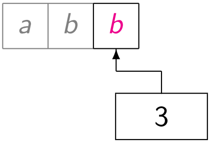

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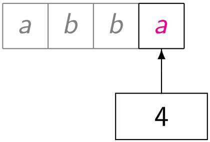

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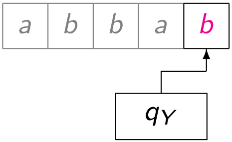

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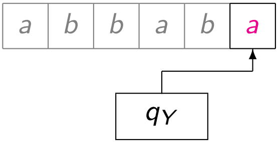

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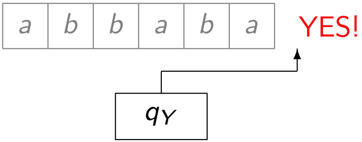

1DFA: $n + 2$ states

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How to locate it?

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Guess Reading the symbol a the automaton can guess that it is the *n*th symbol from the right

Verify In the next steps the automaton verifies such a guess

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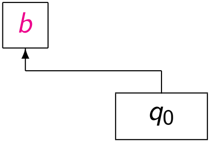

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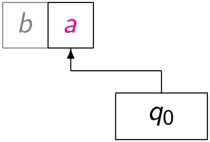

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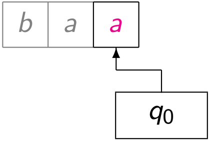

guess
4th symbol from the right

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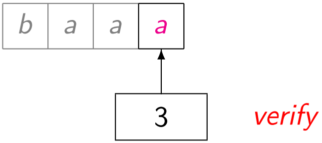

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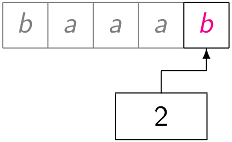

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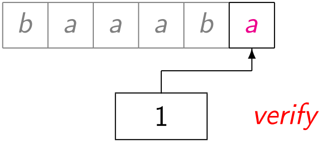

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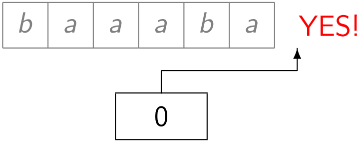

1NFA: $n + 1$ states

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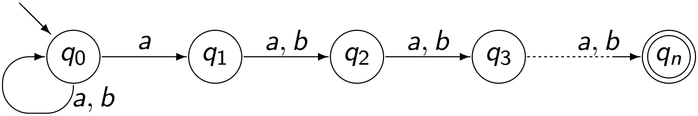

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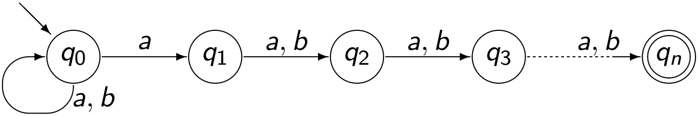

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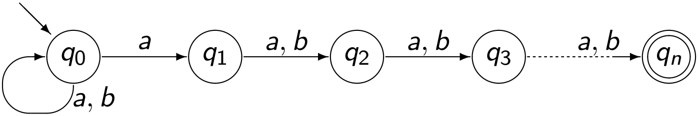

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Remember the previous *n* input symbols!

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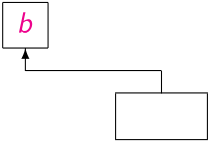

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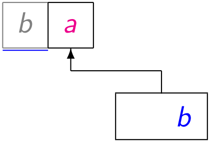

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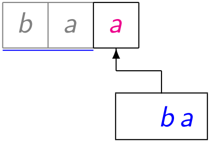

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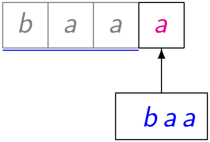

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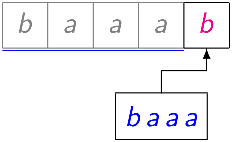

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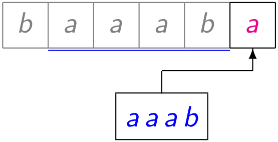

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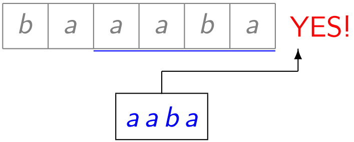

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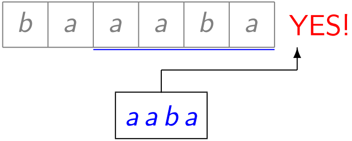

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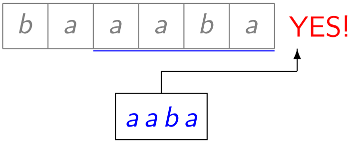

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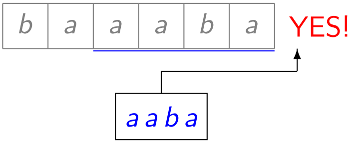

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This is the smallest one!

However...

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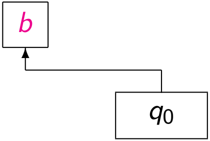

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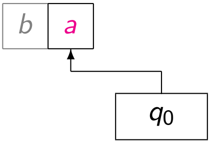

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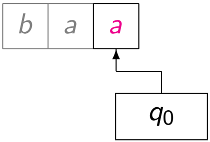

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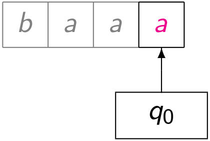

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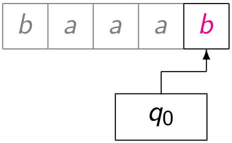

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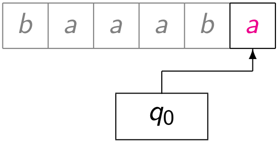

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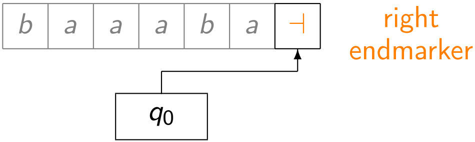

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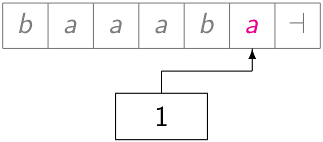

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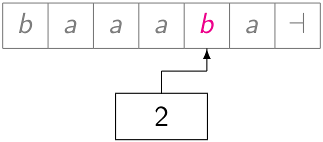

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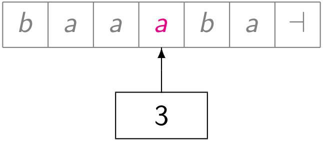

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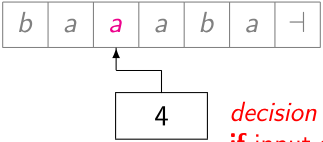

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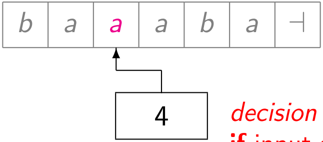

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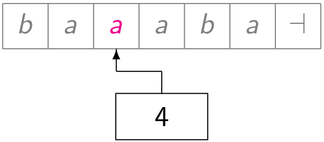

YES!

Two-way deterministic automaton (2DFA): $n + \ldots$ states

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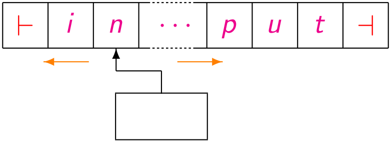
Two-Way Automata: Technical Details



- Input surrounded by the *endmarkers* $\vdash$ and $\dashv$
- Moves
  - to the *left*
  - to the *right*
  - *stationary*
- Initial configuration
- Accepting configuration
- Infinite computations are possible
- *Deterministic* (2DFA) and *nondeterministic* (2NFA) versions

1DFA, 1NFA, 2DFA, 2NFA

What about the power of these models?

1DFA, 1NFA, 2DFA, 2NFA

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They share the same computational power, namely they characterize the class of *regular languages*,

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They share the same computational power, namely they characterize the class of *regular languages*, however...

...some of them are more succinct

Main Example: $L_n = (a+b)^* a (a+b)^{n-1} a (a+b)^*$

1NFA: $n+2$ states

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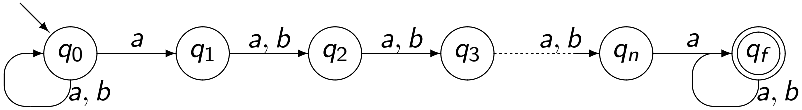

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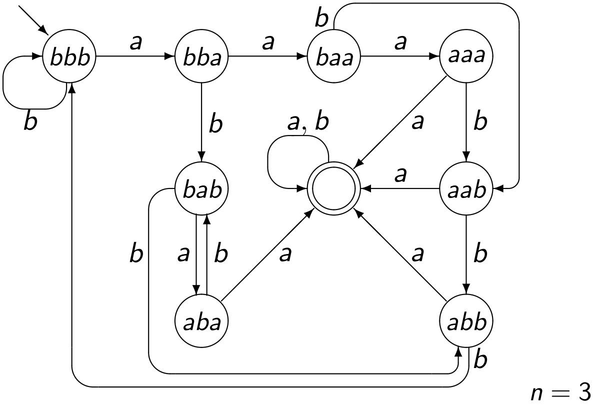

Minimum 1DFA: $2^n + 1$ states

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2DFA ?

Even scanning from the right it seems that
we need to remember a “window” of n symbols

We use a different technique!

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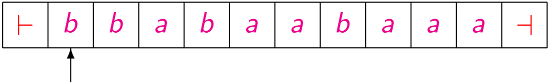

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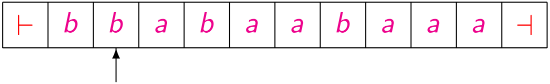

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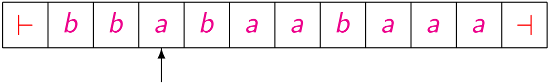

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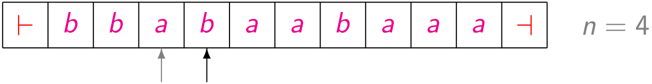

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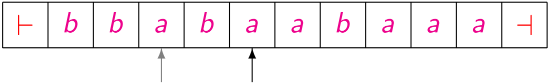

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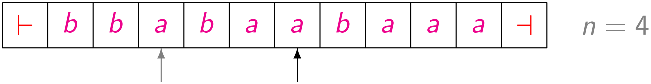

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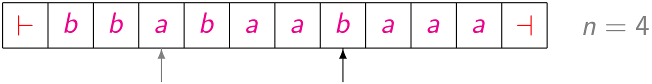

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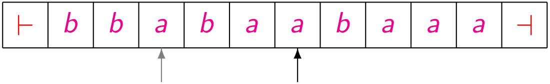

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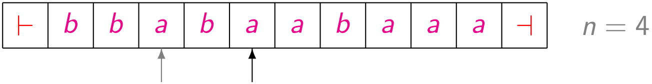

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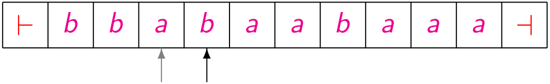

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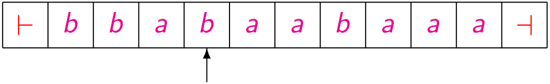

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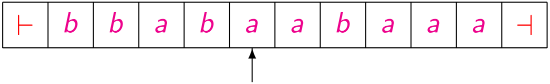

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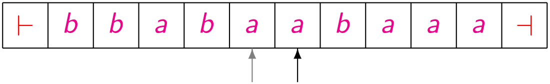

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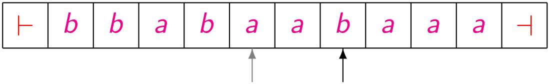

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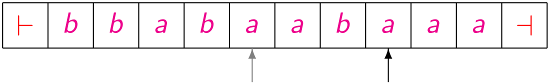

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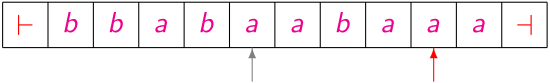

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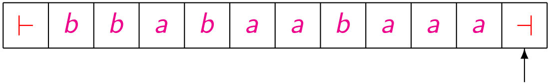

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2DFA: $2n+_{\ldots}$ states

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Summing up,

- L_n is accepted by
  - a 1NFA
  - a 2DFA

  with $O(n)$ states
- Each 1DFA is exponentially larger

Also for this example,
nondeterminism can be removed using two-way motion
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Is it always possible
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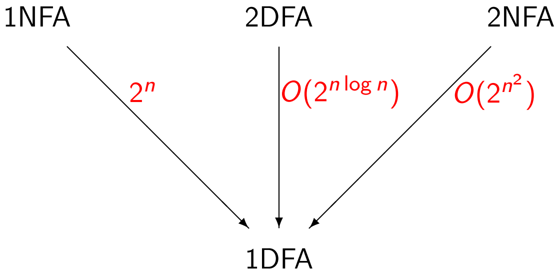
The Question of Sakoda and Sipser

Costs of the Optimal Simulations Between Automata



[Rabin&Scott '59, Shepardson '59, Meyer&Fischer '71, . . .]

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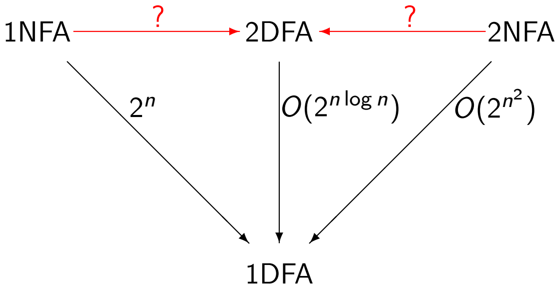

Problem ([Sakoda&Sipser '78])

Do there exist polynomial simulations of

- *1NFAs by 2DFAs*
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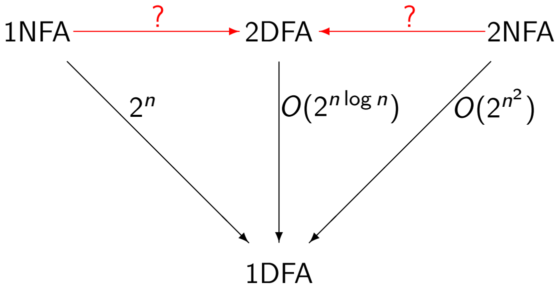

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Do there exist polynomial simulations of

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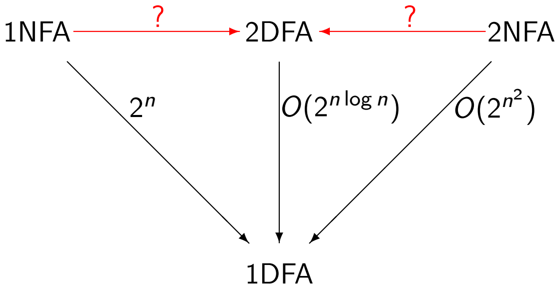
Conjecture

These simulations are not polynomial

Costs of the Optimal Simulations Between Automata



- Exponential upper bounds
  deriving from the simulations of 1NFAs and 2NFAs by 1DFAs
- Polynomial lower bound
  $\Omega(n^2)$ for the cost of the simulation of 1NFAs by 2DFAs
  [Chrobak '86]

Sakoda and Sipser Question

- Very difficult in its general form
- Not very encouraging obtained results:

  Lower and upper bounds too far
  (Polynomial vs exponential)
- Hence:

  Try to attack restricted versions of the problem!

NFAs vs 2DFAs: Restricted Versions

(i) Restrictions on the resulting machines (2DFAs)

- sweeping automata [Sipser '80]
- oblivious automata [Hromkovič&Schnitger '03]
- "few reversal" automata [Kapoutsis '11]

(ii) Restrictions on the languages

- unary regular languages [Geffert Mereghetti&P '03]

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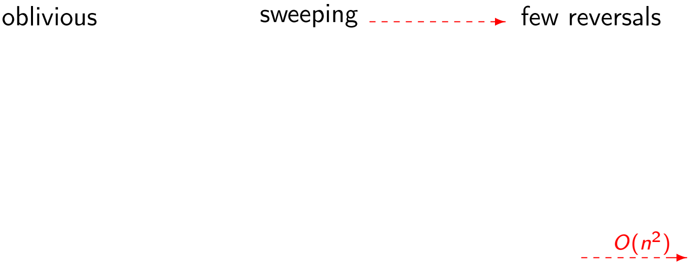
Restrictions on the resulting machines

Restricted Models: Separations

oblivious sweeping few reversals

Restricted Models: Separations



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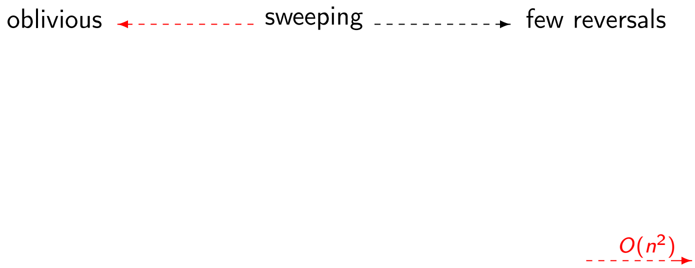

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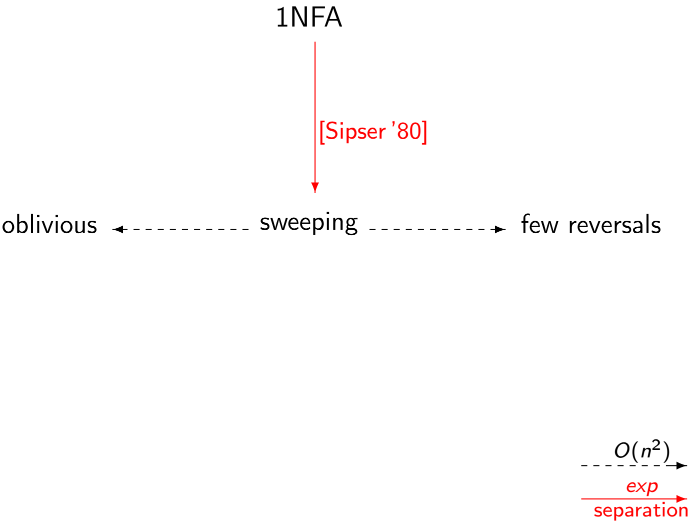

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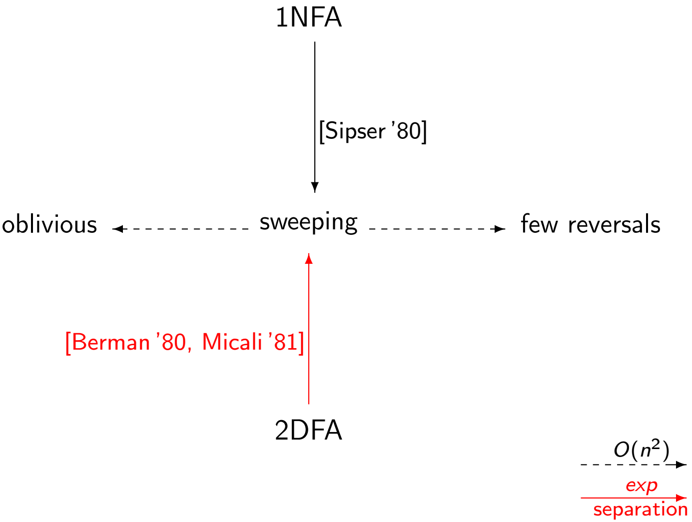

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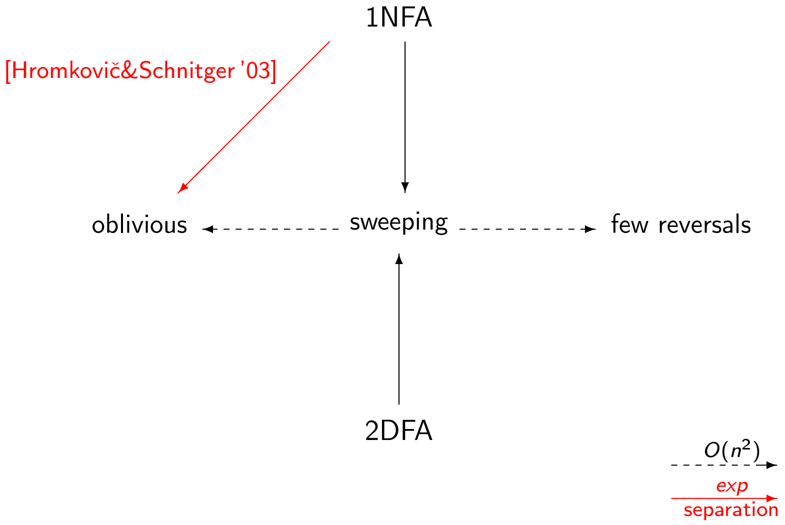

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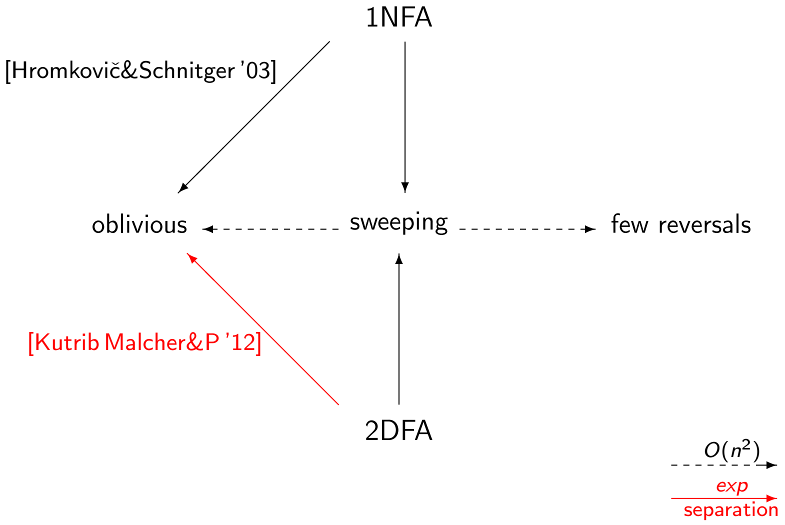

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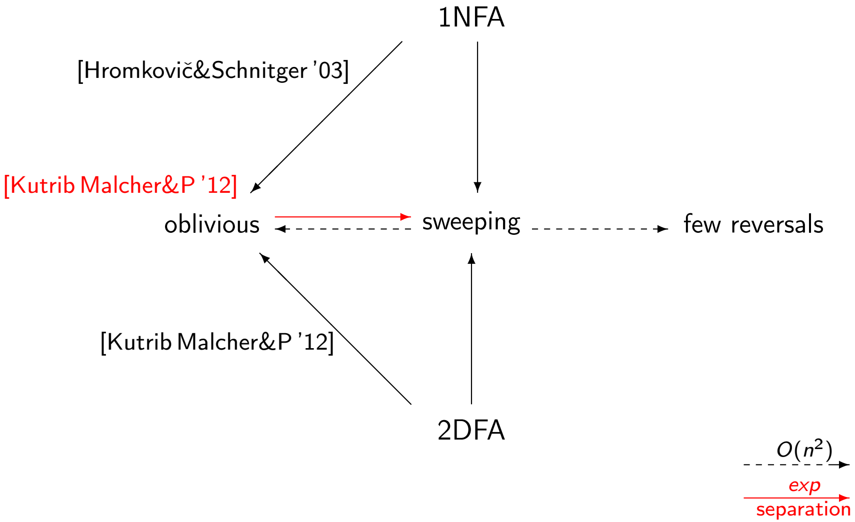
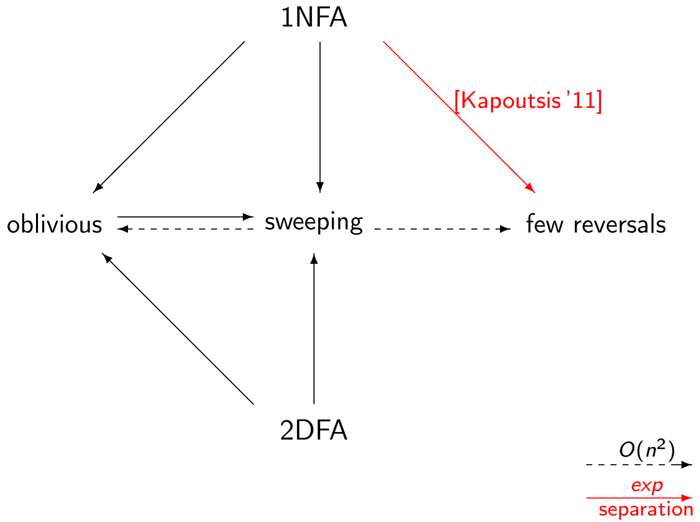


Restricted Models: Separations



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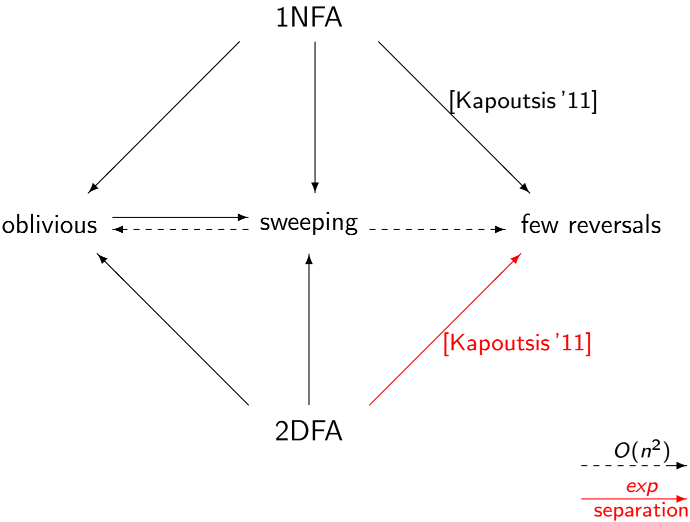

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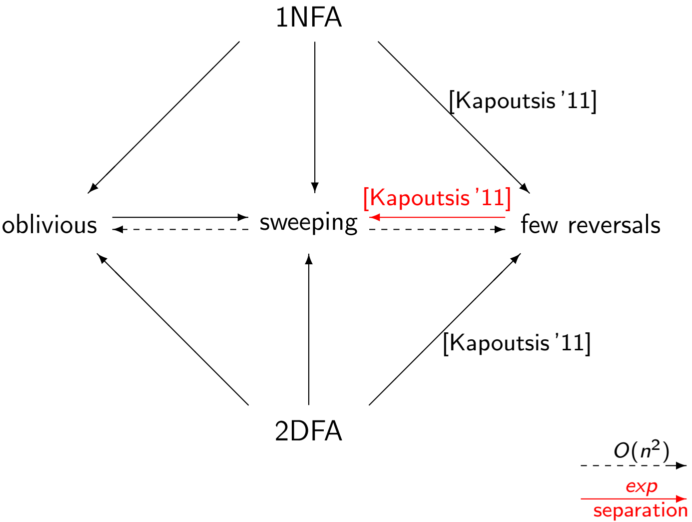

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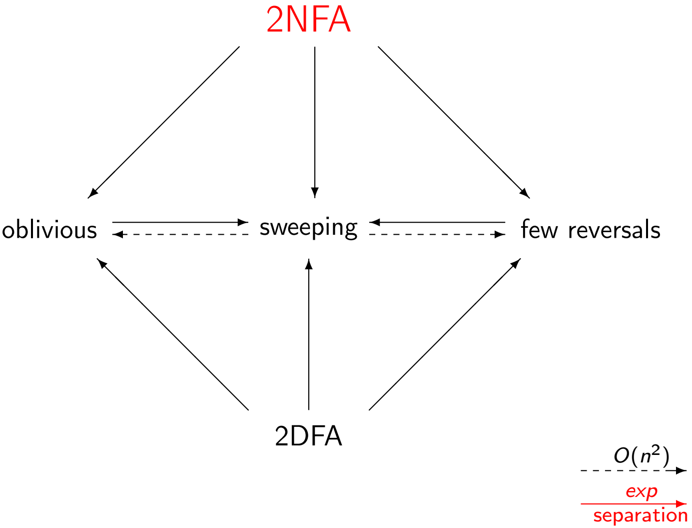

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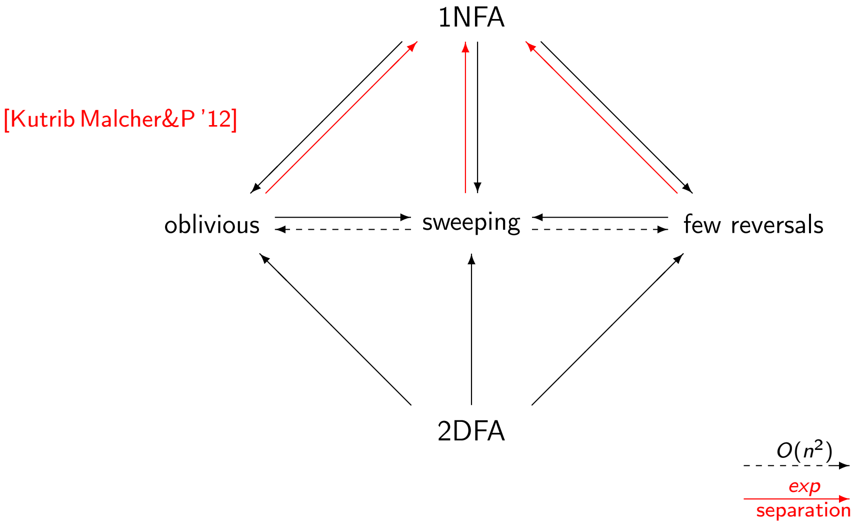

The Unary Case

$$\#\Sigma = 1$$

Optimal Simulation Between Unary Automata

The costs of the optimal simulations between automata are different in the unary and in the general case

1DFA 1NFA

2DFA 2NFA

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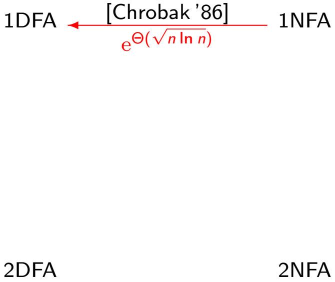

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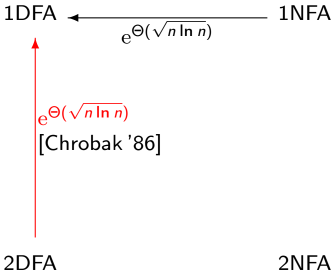

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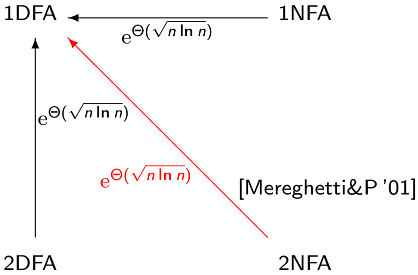

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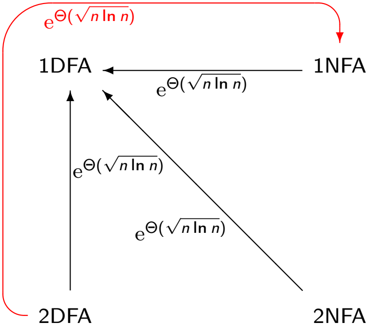

follows from 2DFA $\rightarrow$ 1DFA

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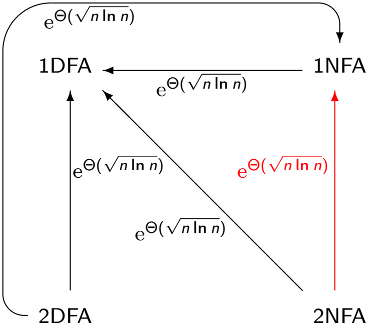

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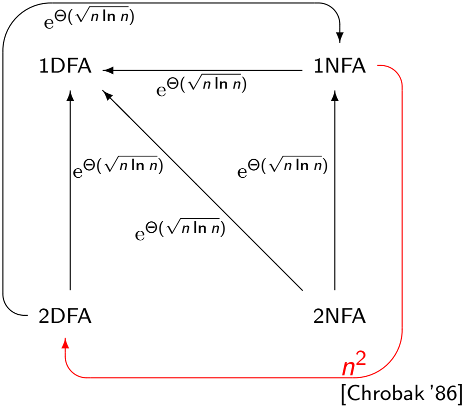

1NFA → 2DFA
In the unary case
this question is solved!
(polynomial conversion)

Optimal Simulation Between Unary Automata

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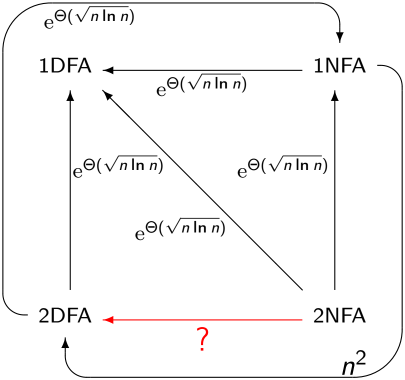

2NFA $\rightarrow$ 2DFA
Even in the unary case
this question is open!

- $e^{\Theta(\sqrt{n \ln n})}$ upper bound (from 2NFA $\rightarrow$ 1DFA)
- $\Omega(n^2)$ lower bound (from 1NFA $\rightarrow$ 2DFA)

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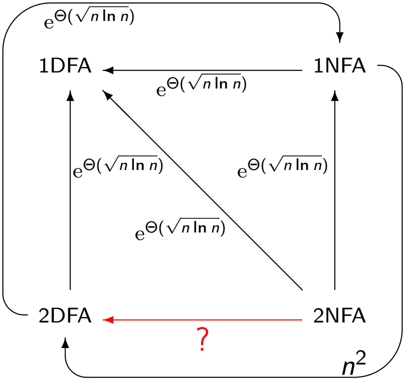

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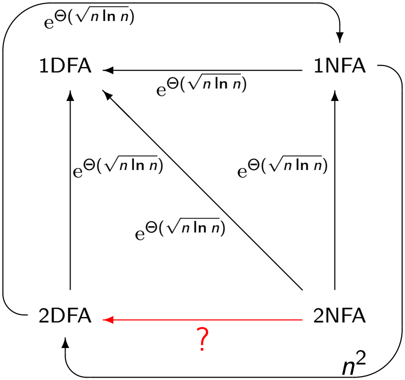

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A better upper bound $e^{O(\ln^2 n)}$ has been proved!

A Normal Form for Unary 2NFAs

Theorem ([Geffert Mereghetti&P '03])

For each unary n-state 2NFA A there exists an almost equivalent 2NFA M s.t.

- *M makes nondeterministic choices and changes the head direction only visiting the end-markers*
- *M has $N \leq 2n+2$ many states*
- *$L(A)$ and $L(M)$ can differ only on strings of length $\leq 5n^2$*

Normal Form for Unary 2NFAs: Consequences

(i) Subexponential simulation of unary 2NFAs by 2DFAs
Each unary n-state 2NFA can be simulated by a 2DFA with $e^{O(\ln^2 n)}$ states [Geffert Mereghetti&P '03]

(ii) Polynomial complementation of unary 2NFAs
Inductive counting argument [Geffert Mereghetti&P '07]

(iii) Polynomial simulation of unary 2NFAs by 2DFAs *under the condition* L = NL [Geffert&P '11]
Hence, proving that the upper bound $e^{O(\ln^2 n)}$ is tight would separate L and NL in the *general case*

(iv) Polynomial simulation of unary 2NFAs by unambiguous 2NFAs *(unconditional)* [Geffert&P '11]

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Outer Nondeterministic Automata (OFAs) [Guillon Geffert&P '12]:

- *nondeterministic choices*

are *possible only* when the head is visiting the *endmarkers*

Hence:

- No restrictions on the *input alphabet*
- No restrictions on *head reversals*
- *Deterministic transitions* on "real" input symbols

Outer Nondeterministic Automata (OFAs)

The results we obtained for the unary case
can be extended to 2OFAs: [Guillon Geffert&P '12]

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Sakoda&Sipser Question: Current Knowledge

- Upper bounds

	1NFA→2DFA	2NFA→2DFA
unary case and OFAs	$O(n^2)$ optimal	$e^{O(\ln^2 n)}$
general case	exponential	exponential

Unary case [Chrobak '86, Geffert Mereghetti&P '03]
OFAs [Guillon Geffert&P '12]

- Lower Bounds
  In all the cases, the best known lower bound is $\Omega(n^2)$ [Chrobak '86]

Conclusion

Final Remarks

- The question of Sakoda and Sipser is very challenging
- In the investigation of restricted versions many interesting and not artificial models have been considered
- The results obtained under restrictions,
  even if not solving the full problem,
  are not trivial and, in many cases, very deep
- Connections with space and structural complexity
  - questions
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Thank you for your attention!