# Strongly Limited Automata


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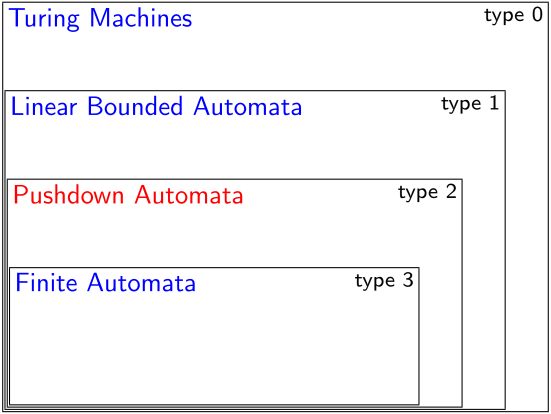
DI MILANO

# The Chomsky Hierarchy



# Limited Automata [Hibbard'67]

One-tape Turing machines with restricted rewritings

**Definition**

Fixed an integer $d \geq 1$, a *$d$-limited automaton* is

- a one-tape Turing machine
- which is allowed to rewrite the content of each tape cell *only in the first $d$ visits*

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## Computational power

- For each $d \geq 2$, $d$-limited automata characterize context-free languages [Hibbard'67]
- 1-limited automata characterize regular languages [Wagner&Wechsung'86]

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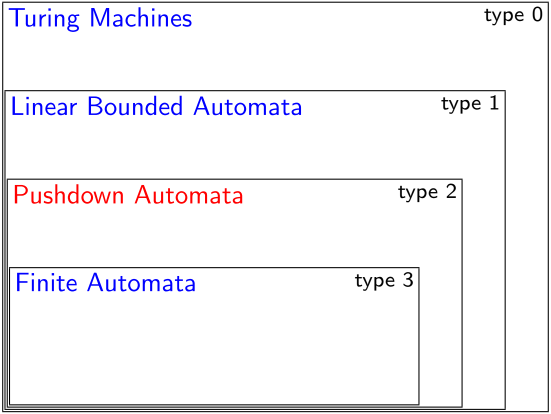

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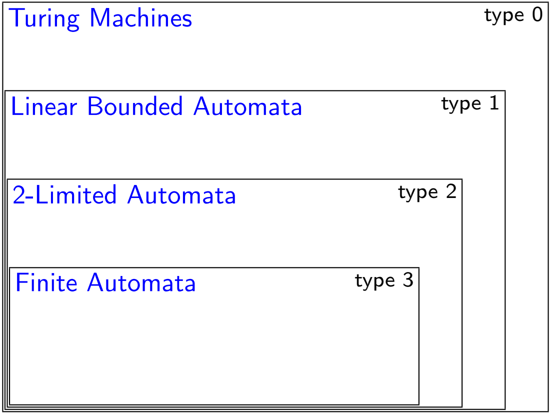

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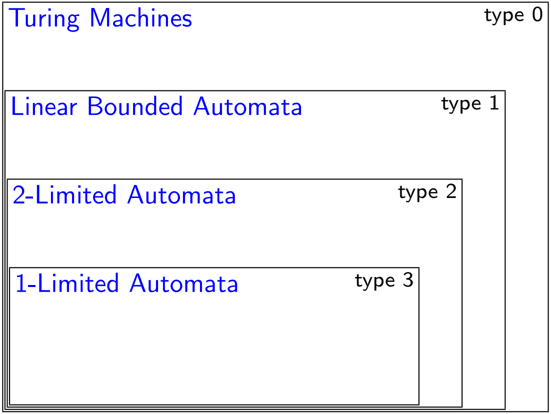

# Motivations

- Dyck languages are accepted without fully using capabilities of 2-limited automata
- Chomsky-Schützenberger Theorem: Recognition of CFLs can be reduced to recognition of Dyck languages

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*Is it possible to restrict 2-limited automata without affecting their computational power?*

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**Question**

*Is it possible to restrict 2-limited automata without affecting their computational power?*

YES!

**Forgetting Automata**
[Jancar&Mráz&Plátek '96]

- The content of any cell can be erased in the 1st or 2nd visit (using a fixed symbol)
- No other changes of the tape are allowed

# A Different Restriction: Strongly Limited Automata

- Model inspired by the algorithm used by 2-limited automata to recognize Dyck languages
- Restrictions on
  - state changes
  - head reversals
  - rewriting operations
- Computational power: same as 2-limited automata (CFLs)
- Descriptional power: the sizes of equivalent
  - CFGs
  - PDAs
  - strongly limited automata

  are polynomially related

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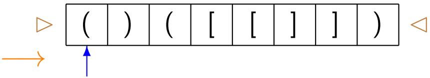


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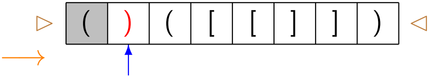


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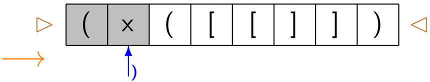


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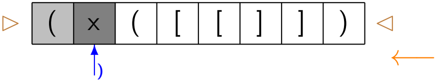


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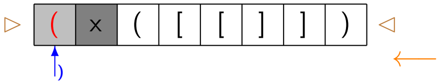


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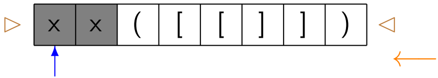


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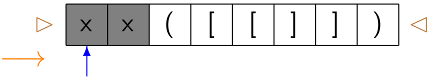


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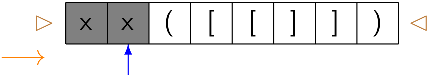


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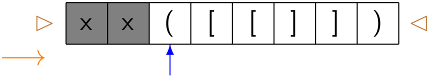


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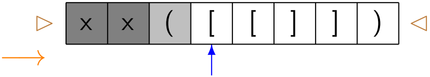


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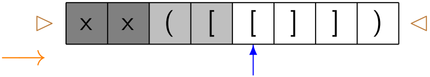


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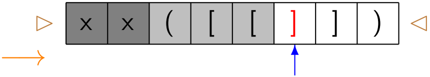


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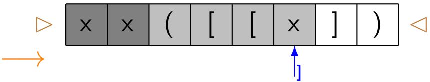


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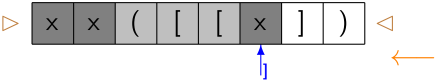


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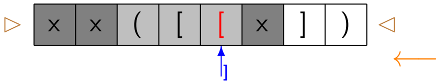


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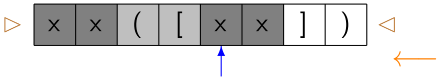


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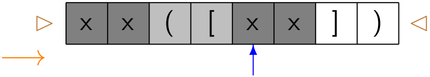


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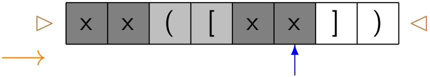


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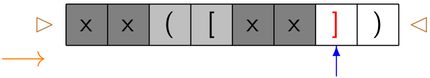


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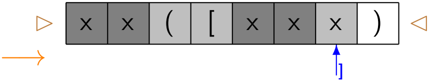


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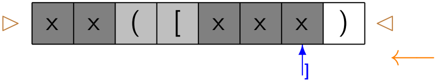


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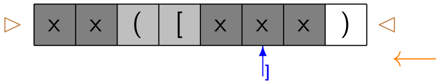


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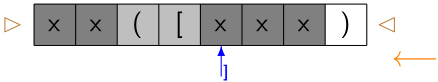


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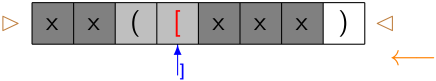


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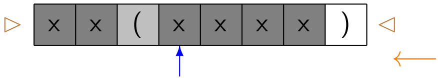


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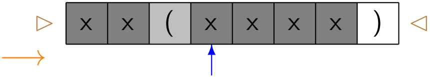


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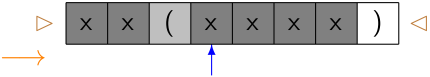


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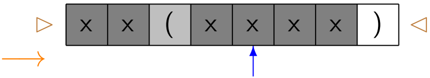


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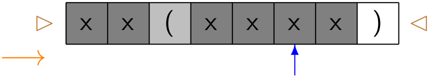


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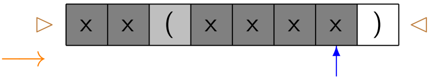


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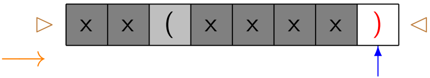


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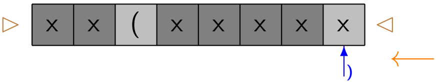


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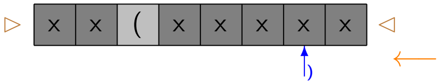


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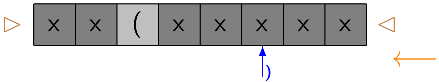


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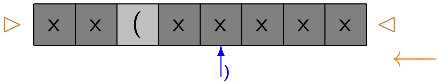


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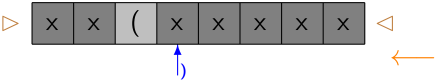


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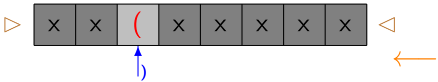


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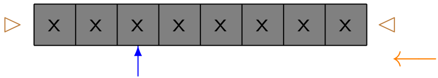


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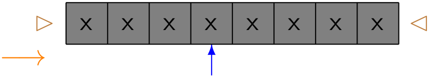


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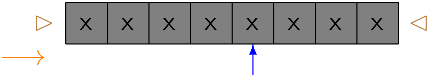


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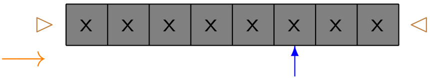


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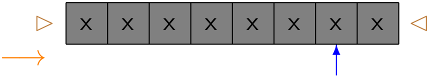


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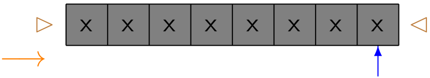


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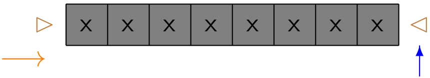


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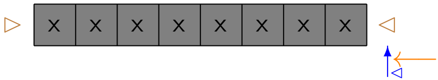


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Special cases:

(i') When ◁ is reached scan all the tape
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(iii') If in (iii) ▷ is reached then *reject*

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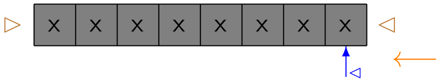


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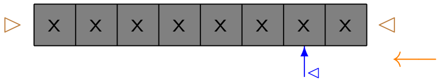


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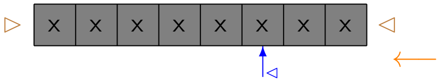


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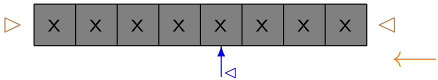


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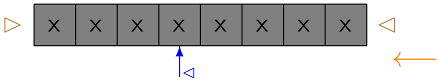


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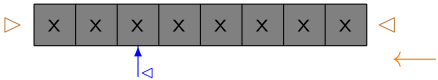

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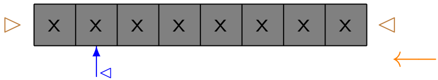


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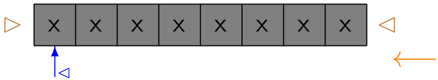


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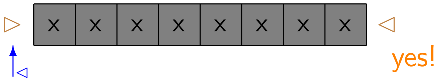


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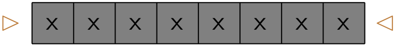


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# Dyck Language Recognition

$\triangleright$ | x | x | x | x | x | x | x | x | $\triangleleft$

(i) Move to the right to search a closed bracket

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Special cases:

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# Dyck Language Recognition

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ▷ | ( | ) | ( | [ | [ | ] | ] | ) | ◁ |

- Moves to the right:
  - to search a closed bracket
- Moves to the left:
  - to search an open bracket
  - to check the tape content in the final scan from right to left
- Rewritings:
  - each closed bracket is rewritten in the first visit
  - each open bracket is rewritten in the second visit
  - no rewritings in the final scan

# Dyck Language Recognition

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\triangleright$ | ( | ) | ( | [ | [ | ] | ] | ) | $\triangleleft$ |

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|---|---|---|---|---|---|---|---|---|---|

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Only one state $q_0$!

- Moves to the left:
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    One state for each type of bracket!
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# Dyck Language Recognition

$$\triangleright \; \boxed{(}\boxed{)}\boxed{(}\boxed{[}\boxed{[}\boxed{]}\boxed{]}\boxed{)} \; \triangleleft$$

- Moves to the right:
  - to search a closed bracket — Only one state $q_0$!
- Moves to the left:
  - to search an open bracket — One state for each type of bracket!
  - to check the tape content in the final scan from right to left
- Rewritings:
  - each closed bracket is rewritten in the first visit
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  - no rewritings in the final scan

# Extended Dyck Language

- Strings padded with "neutral symbols"

$$| \quad | \quad [ \quad | \quad ( \quad | \quad ) \quad ] \quad | \quad ( \quad ) \quad |$$

# Extended Dyck Language

- Strings padded with "neutral symbols"
- Similar recognition technique:
  - while moving to the left searching an open bracket, neutral symbols are rewritten
  - the tape should finally contain only neutral symbols and x's

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ▷ | $\vert$ | $\vert$ | [ | $\vert$ | ( | $\vert$ | ) | ] | $\vert$ | ( | ) | $\vert$ | ◁ |

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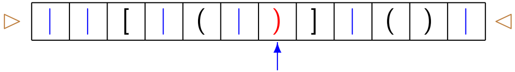

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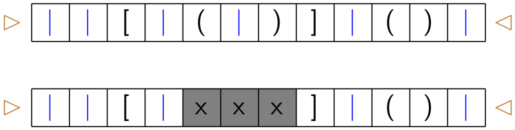

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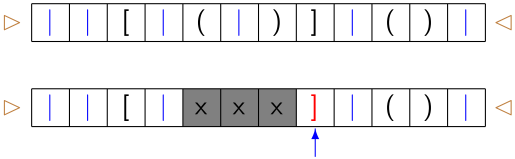

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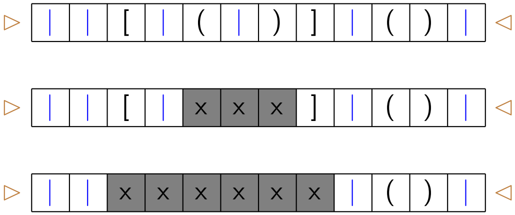

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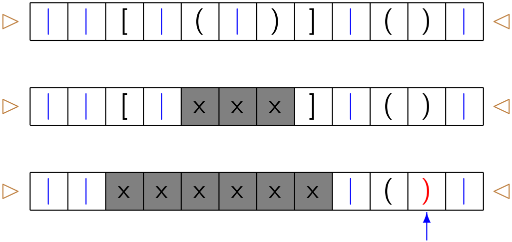

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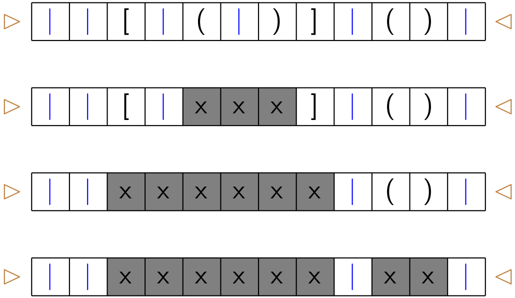

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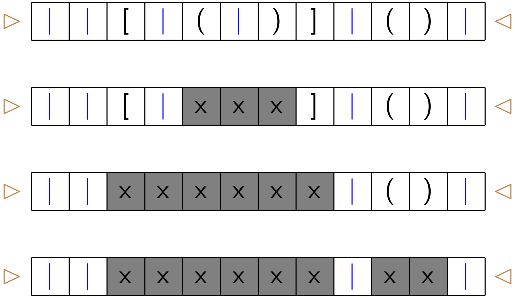


- The procedure can be adapted *to generate* strings in the language

# Strongly Limited Automata

- Alphabet
  - $\Sigma$ input
  - $\Gamma$ working
  - $\Upsilon = \Sigma \cup \Gamma \cup \{\rhd, \lhd\}$ global alphabet
- States and moves

# Strongly Limited Automata

- Alphabet

  $\Sigma$ input

  $\Gamma$ working

  $\Upsilon$ $= \Sigma \cup \Gamma \cup \{\triangleright, \triangleleft\}$ global alphabet

- States and moves

  $q_0$ initial state, moving from left to right

  $\dashrightarrow$ *move to the right*

  $q \xleftarrow{X}$ write $X \in \Gamma$, enter state $q \in Q_L$, *turn to the left*

# Strongly Limited Automata

$\rhd$ | $|$ | ( | ) | ( | [ | ] | $|$ | ) | [ | ] | $\lhd$

$\dots$

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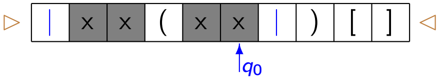


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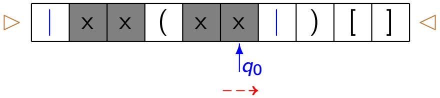


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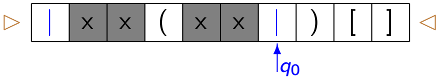


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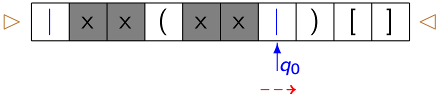


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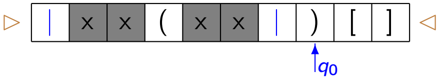


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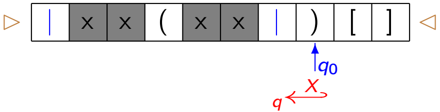


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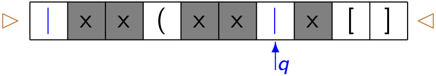


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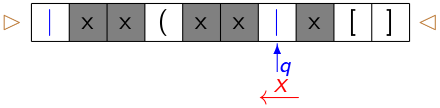
# Strongly Limited Automata

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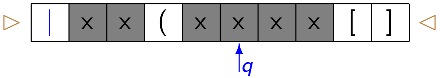


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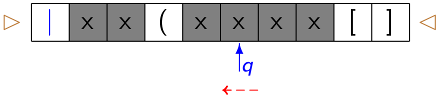


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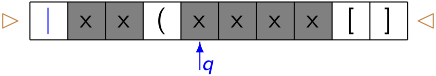


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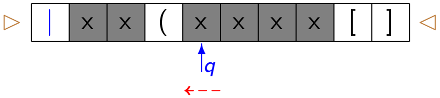


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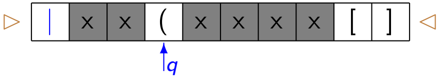


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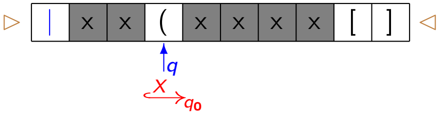


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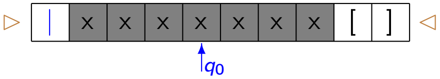


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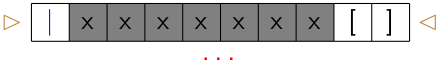


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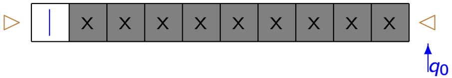


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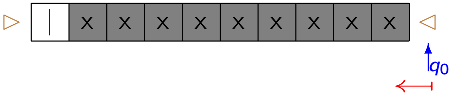


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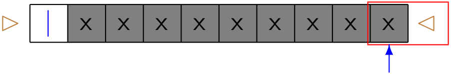


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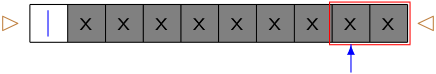


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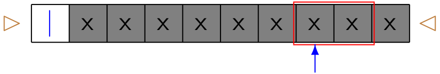


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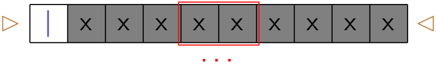

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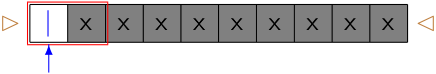

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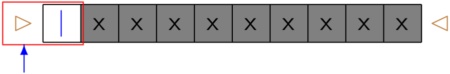


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# A Variant of the Chomsky-Schützenberger Theorem

$\Omega_{k,\ell}$ alphabet with $k$ types of brackets and $\ell$ neutral symbols

$\widehat{D}_{k,\ell}$ *extended* Dyck language over $\Omega_{k,\ell}$

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**Theorem ([Okhotin'12])**

*$L \subseteq \Sigma^*$ is context-free iff there exist*

- *integers $k, \ell \geq 1$*
- *a regular language $R \subseteq \Omega^*_{k,\ell}$*
- *a letter-to-letter homomorphism $h : \Omega_{k,\ell} \to \Sigma$*

*such that $L = h(\widehat{D}_{k,\ell} \cap R)$*

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Remarks

- $k, \ell$ are *polynomial* wrt the size of each CFG specifying $L$
- The language $R$ is *local*

# From CFLs to Strongly Limited Automata

$\triangleright$ | $a$ | $b$ | $b$ | $a$ | $a$ | $b$ | $b$ | $a$ | $a$ | $\triangleleft$

$L \subseteq \Sigma^*$ given CFL

$w \in \Sigma^*$ input string

$w \in L$?

# From CFLs to Strongly Limited Automata

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\triangleright$ | $a$ | $b$ | $b$ | $a$ | $a$ | $b$ | $b$ | $a$ | $a$ | $\triangleleft$ |

$L \subseteq \Sigma^*$ given CFL

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$L = h(\widehat{D}_{k,\ell} \cap R)$

Strongly limited automaton $M$ for $L$:

- Guess and write on the tape $x \in \widehat{D}_{k,\ell}$
- While guessing each symbol $x_i$, check if $h(x_i) = w_i$
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# From CFLs to Strongly Limited Automata

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|---|---|---|---|---|---|---|---|---|---|---|

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|---|---|---|---|---|---|---|---|---|---|---|
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$L \subseteq \Sigma^*$ given CFL

$w \in L$?

$\triangleright$ | a | b | b | a | a | b | b | a | a | $\triangleleft$ $\quad w \in \Sigma^*$ input string

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Given a CFG $G$ for $L$, the size of $M$ is polynomial in the size of $G$

# From CFLs to Strongly Limited Automata

$L \subseteq \Sigma^*$ given CFL

$w \in L$?

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\triangleright$ | $a$ | $b$ | $b$ | $a$ | $a$ | $b$ | $b$ | $a$ | $a$ | $\triangleleft$ |

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$h(x) = w$? $\quad x \in R$?

Strongly limited automaton $M$ for $L$:

- Guess and write on the tape $x \in \widehat{D}_{k,\ell}$
- While guessing each symbol $x_i$, check if $h(x_i) = w_i$
- In the final scan checks if $x \in R$

Given a CFG $G$ for $L$, the size of $M$ is polynomial in the size of $G$

**CFGs $\rightarrow$ Strongly Limited Automata**

Polynomial size!

# Simulation of Strongly Limited Automata by PDAs

The simulation of 2-limited automata by PDAs is *exponential* in the description size [P&Pisoni'13]

**Problem**

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Tape cell $c$ reached for the first time:

$\dashrightarrow$ content not modified now, but
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guess the symbol written in the 2nd visit and
save it on the stack with the current symbol

${}_q\!\xleftarrow{X}$ content modified, head turned to the left

enter *back mode* to check previous guesses
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Visits after 1st rewriting:
no changes of content and state

These visits do not need to be simulated

Final scan (from right to left)

Simulated from left to right
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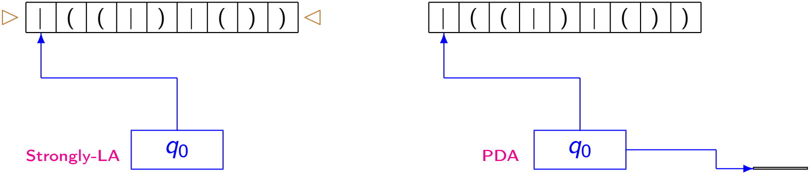

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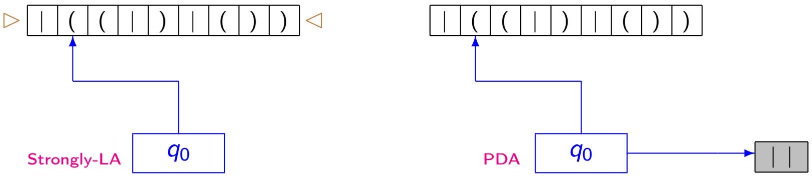

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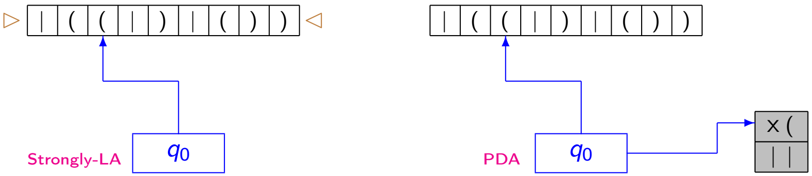

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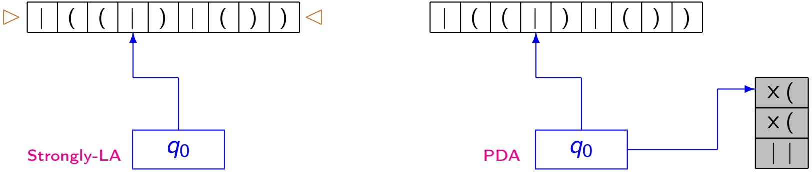

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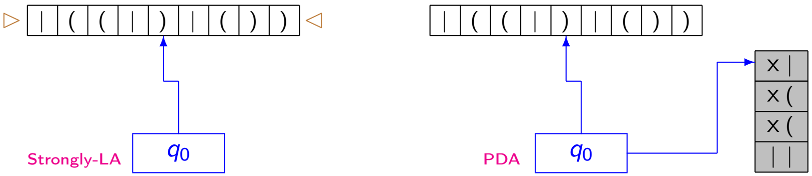

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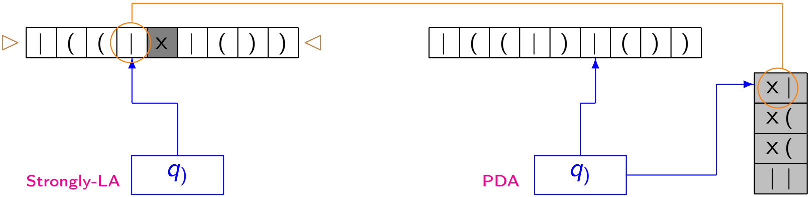

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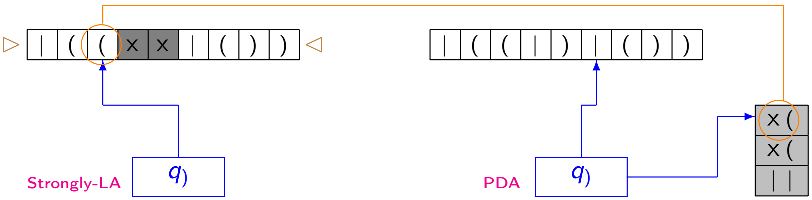

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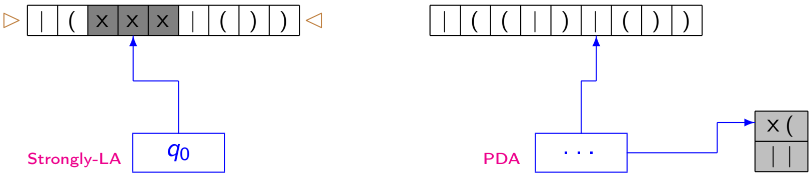

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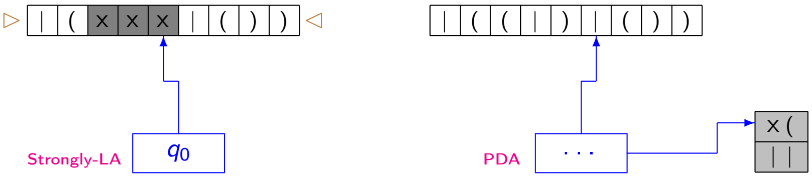

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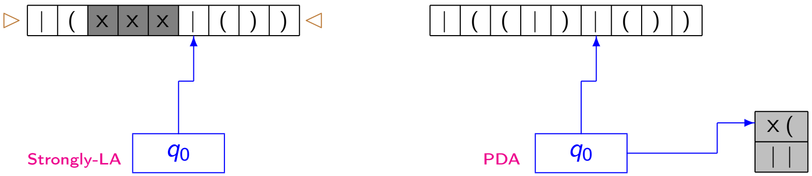

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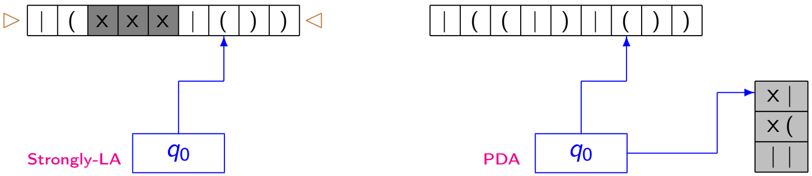

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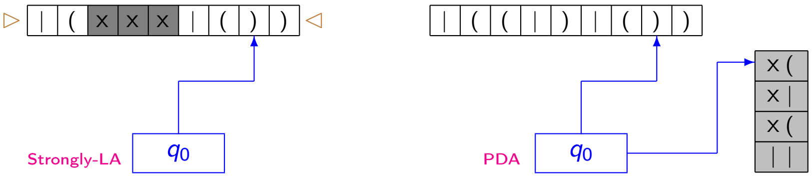

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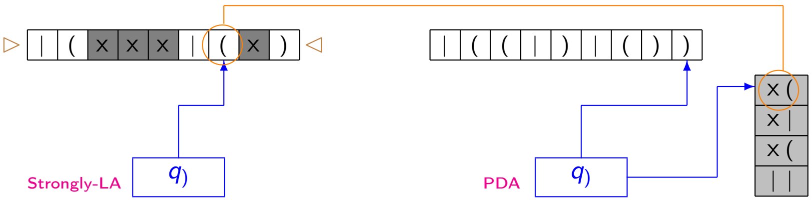

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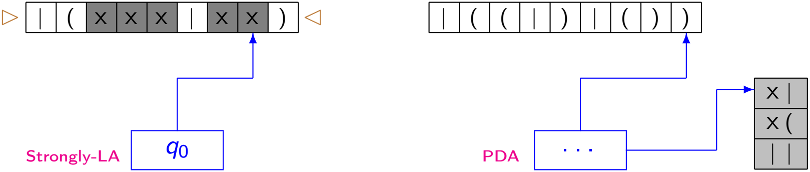

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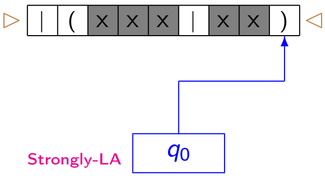


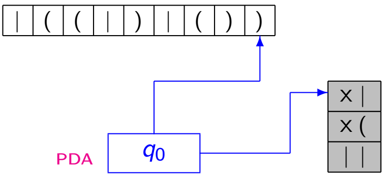

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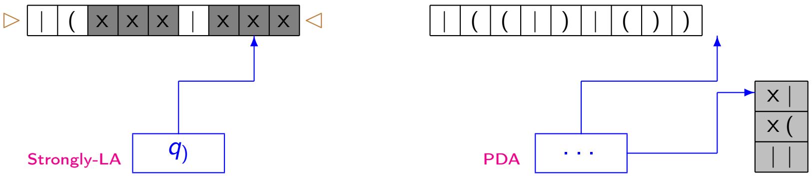

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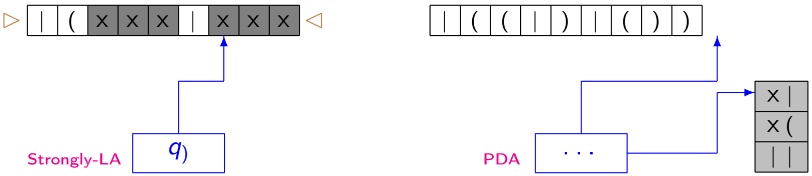

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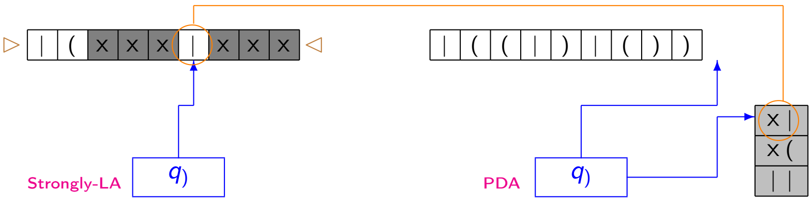

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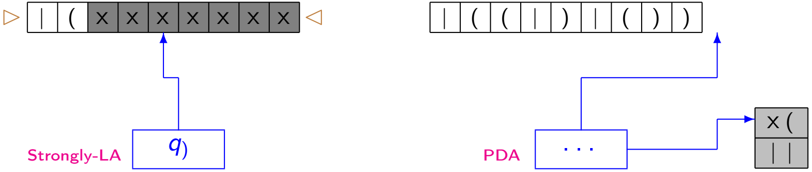

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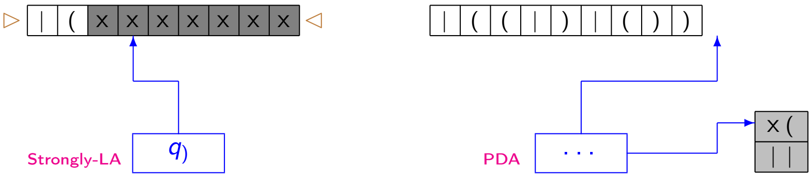

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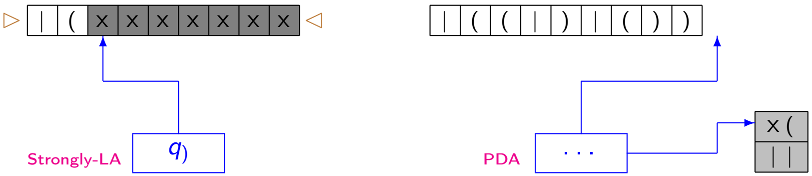

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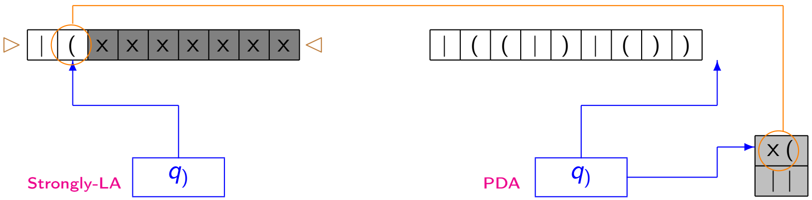

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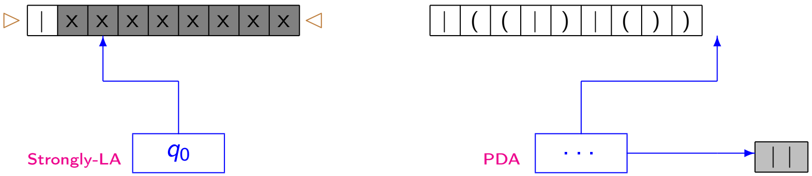

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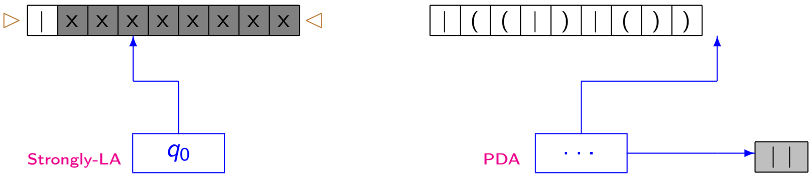

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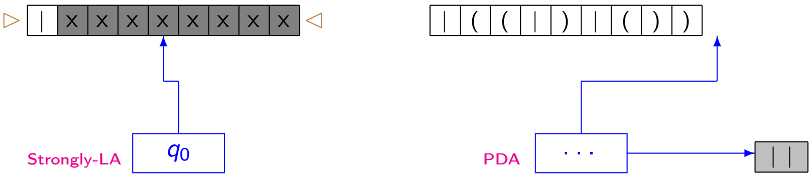

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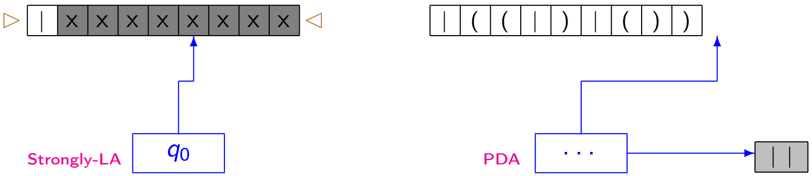

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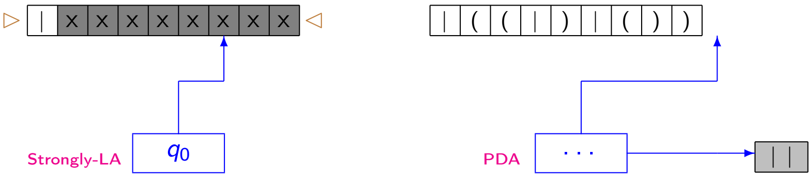

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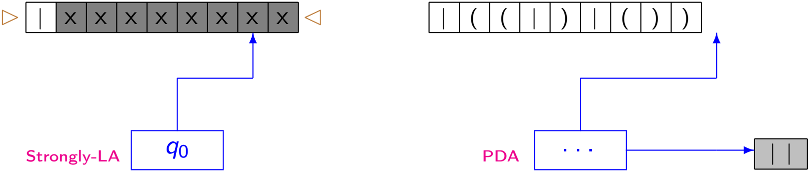

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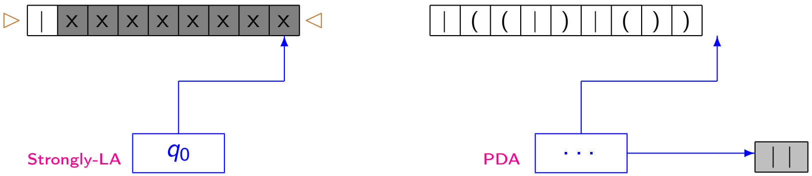

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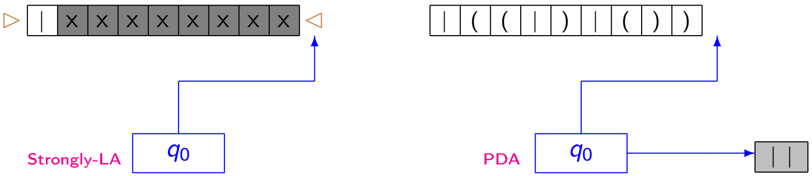

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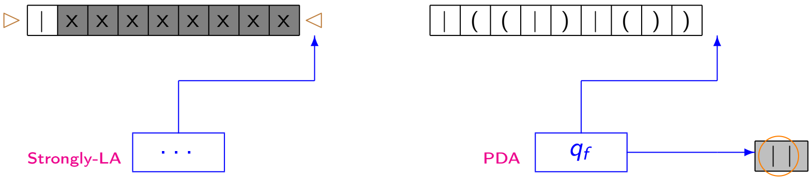

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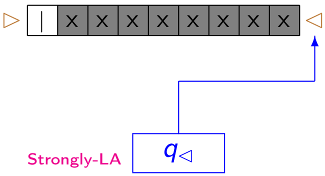
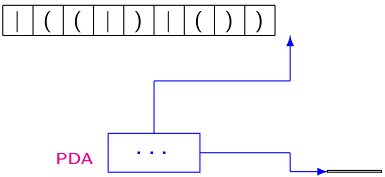



final scan from right to left




final scan already simulated

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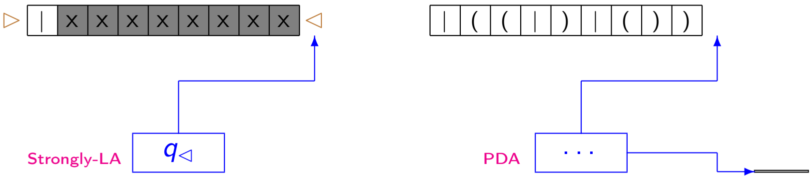


The description of the resulting PDA has polynomial size wrt that of the given strongly limited automaton

# Summing up...

- Descriptional complexity
  - Strongly limited automata
  - Context-free grammars
  - Pushdown automata

  are polynomially related in size

- 2-limited automata can be exponentially smaller [P&Pisoni'13]

# Strongly Limited Automata vs Forgetting Automata

- Strongly limited automata can use different symbols to rewrite tape cells, e.g.,
  $\{ww^R \mid w \in \{a,b\}^*$ does not contain two consecutive $b$s$\}$

**Problem**

*Which class of languages is accepted by strongly limited automata that can use only one fixed symbol for rewriting?*

- Forgetting Automata [Jancar&Mráz&Plátek '96]:
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- The conversion from CFGs to strongly limited automata uses nondeterminism
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$$L_1 = \{ca^n b^n \mid n \geq 0\} \cup \{da^{2n} b^n \mid n \geq 0\}$$
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