

Limited Automata and Context-Free Languages

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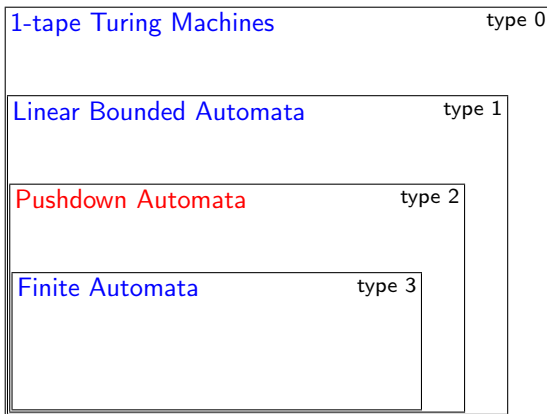
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The Chomsky Hierarchy



Limited Automata [Hibbard'67]

One-tape Turing machines with restricted rewritings

Definition

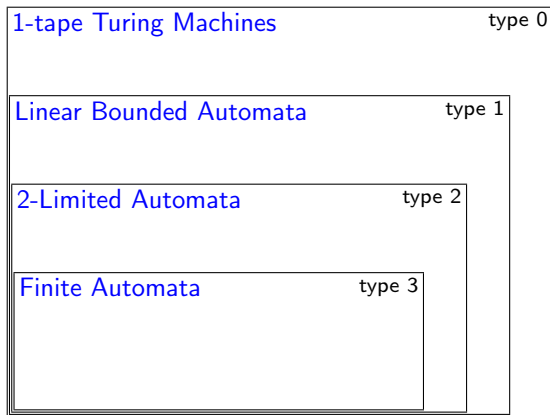
Fixed an integer $d \geq 1$, a *d-limited automaton* is

- ▶ a one-tape Turing machine
- ▶ which is allowed to rewrite the content of each tape cell *only in the first d visits*

Computational power

- ▶ For each $d \geq 2$, *d-limited automata* characterize context-free languages [Hibbard'67]
- ▶ 1-limited automata characterize regular languages [Wagner&Wechsung'86]

The Chomsky Hierarchy



Our Contributions

- ▶ 2-Limited Automata \equiv Pushdown Automata:
descriptive complexity point of view

2-LAs \rightarrow PDAs

Exponential gap

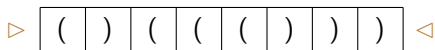
PDAs \rightarrow 2-LAs

Polynomial upper bound

- ▶ Determinism vs Nondeterminism

Deterministic Context-Free Languages \equiv Deterministic 2-LAs

Example: Balanced Parentheses



- (i) Move to the right to search a closed parenthesis
- (ii) Rewrite it by #
- (iii) Move to the left to search an open parenthesis
- (iv) Rewrite it by #
- (v) Repeat from the beginning

Special cases:

- (i') If in (i) the right end of the tape is reached then scan all the tape and *accept* iff all tape cells contain #
- (iii') If in (iii) the left end of the tape is reached then *reject*

Each cell is rewritten only in the first 2 visits!

Simulation of 2-Limited Automata by Pushdown Automata

Problem

How much it costs, in the description size, the simulation of 2-LAs by PDAs?

This work

Exponential cost!

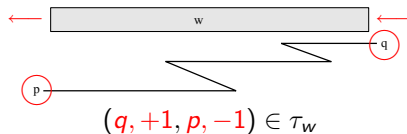
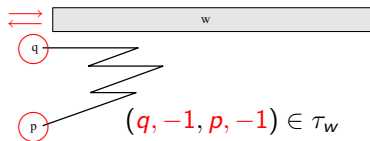
Transition Tables of 2-LAs

► Fixed a 2-limited automaton

► *Transition table* τ_w

w is a “frozen” string

$$\tau_w \subseteq Q \times \{-1, +1\} \times Q \times \{-1, +1\}$$

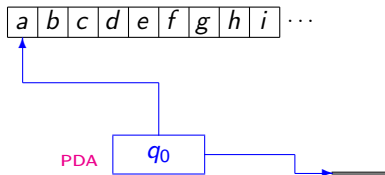
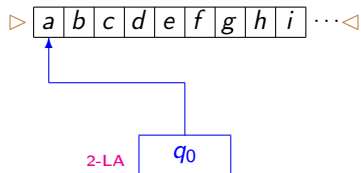


$(q, d', p, d'') \in \tau_w$ iff M on a tape segment containing w has a computation path:

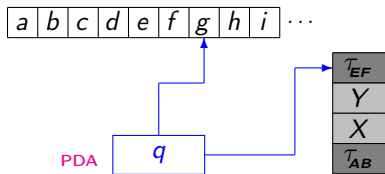
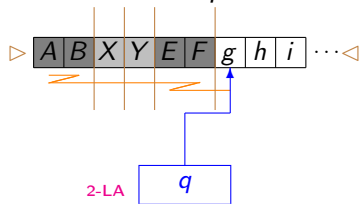
- entering the segment in q from d'
- exiting the segment in p from d''
- left = -1 , right = $+1$

Simulation of 2-LAs by PDAs

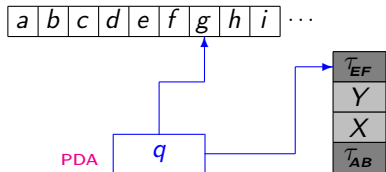
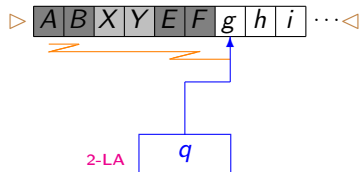
Initial configuration



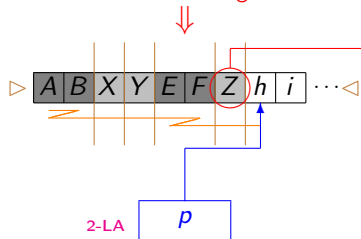
After some steps...



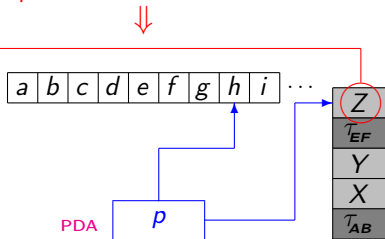
Simulation of 2-LAs by PDAs



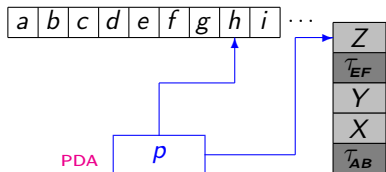
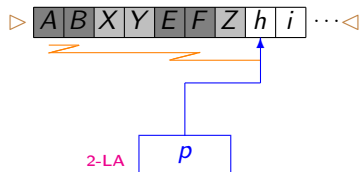
$\delta(q, g) \ni (p, Z, +1)$
 move to the right



normal mode
 push and direct simulation

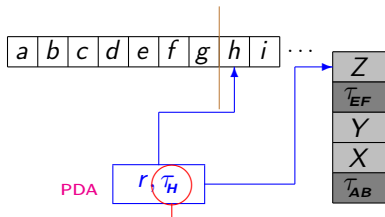
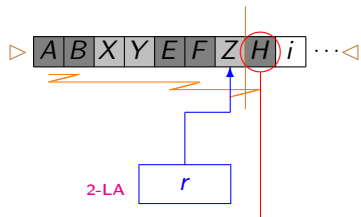


Simulation of 2-LAs by PDAs

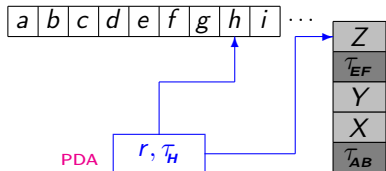
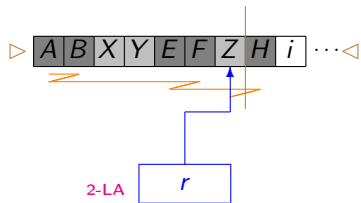


$\delta(p, h) \ni (r, H, -1)$
 move to the left

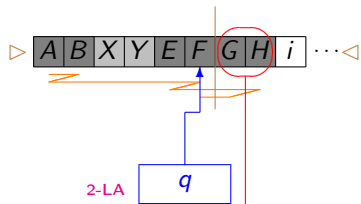
back mode



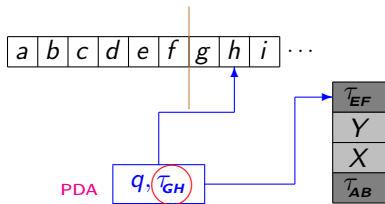
Simulation of 2-LAs by PDAs



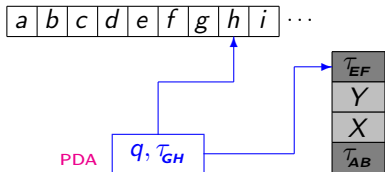
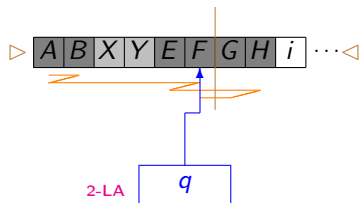
$\delta(r, Z) \ni (q, G, -1)$
move to the left



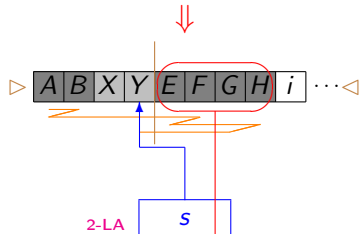
back mode



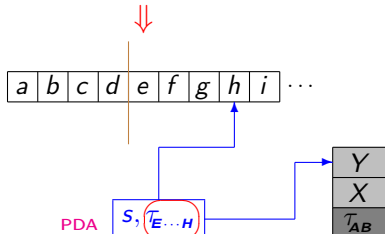
Simulation of 2-LAs by PDAs



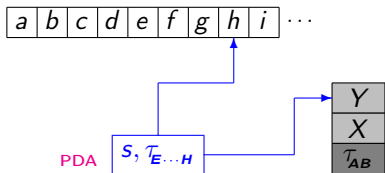
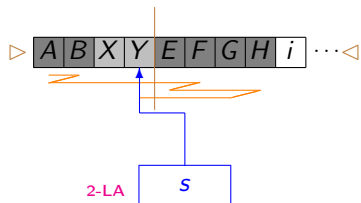
$(q, +1, s, -1) \in \tau_{EF}$
exit to the left



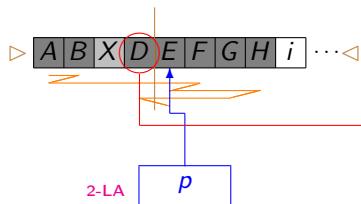
back mode



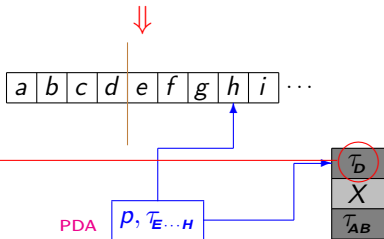
Simulation of 2-LAs by PDAs



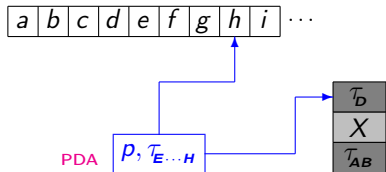
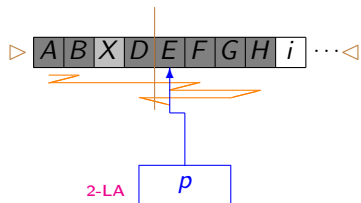
$\delta(s, Y) \ni (p, D, +1)$
move to the right



back mode

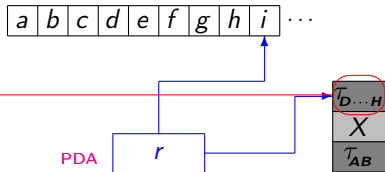
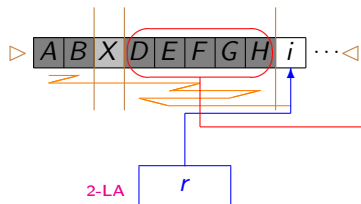


Simulation of 2-LAs by PDAs



$(p, -1, r, +1) \in \tau_{E...H}$
exit to the right

resume normal mode
move to the right



Simulation of 2-LAs by PDAs

Summing up...

Cost of the simulation

- ▶ In the resulting PDA transition tables are used for
 - states
 - pushdown alphabet
- ▶ *Exponential upper bound* for the size of the resulting PDA
- ▶ Optimal

Determinism vs nondeterminism

- ▶ Determinism is preserved by the simulation
provided that the input of the PDA is right end-marked
- ▶ *Double exponential* size for the simulation of D2-LAs by DPDA
- ▶ Conjecture: this cost cannot be reduced

Simulation of Pushdown Automata by 2-Limited Automata

PDA_s → 2-LA_s

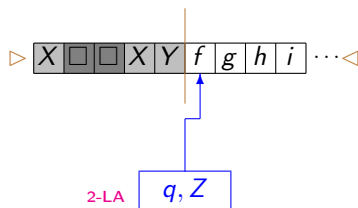
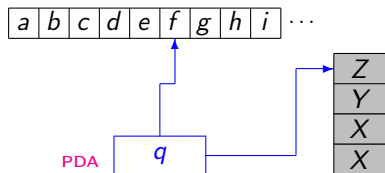
Polynomial cost!

DPDA_s → D2-LA_s

Polynomial cost!

(in the description size)

Simulation of PDAs by 2-LAs



Normal form for (D)PDAs:

- ▶ at each step, the stack height increases at most by 1
- ▶ ϵ -moves cannot push on the stack

Each (D)PDA can be simulated by an equivalent (D)2-LA of polynomial size

Determinism vs Nondeterminism in Limited Automata

Corollary of the simulations

Deterministic 2-LAs \equiv Deterministic Context-Free Languages

On the other hand, the language

$$L = \{a^n b^n c \mid n \geq 0\} \cup \{a^n b^{2^n} d \mid n \geq 0\}$$

is accepted by a *deterministic* 3-LA, but it is not a DCFL

Infinite hierarchy [Hibbard'67]

For each $d \geq 2$ there is a language which is accepted by a deterministic d -limited automaton and that cannot be accepted by any deterministic $(d - 1)$ -limited automaton

Futher Investigations

- ▶ Descriptive complexity aspects for $d > 2$

We conjecture that for $d > 2$ the size gap from d -limited automata to PDAs remains exponential

- ▶ Descriptive complexity aspects in the unary case

- Unary context-free languages are regular [Ginsburg&Rice'62]

- Ex: $L_n = (a^{2^n})^*$

	size
2-LA	$O(n)$
DPDA	$O(n)$
minimal DFA	2^n
minimal 2NFA	2^n

Thank you for your attention!