# Two-Way Automata Making Choices Only at the Endmarkers


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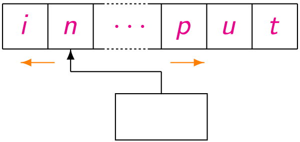
LATA 2012 – A Coruña, Spain – March 7, 2012

# Finite State Automata




Base versions:

- one-way deterministic (1DFA)
- one-way nondeterministic (1NFA)

Possibile variants:

- *two-way automata:* input head moving forth and back
  - 2DFA
  - 2NFA
- alternating automata
- ...

# 1DFA, 1NFA, 2DFA, 2NFA

What about the power of these models?

They share the same computational power, namely they characterize the class of *regular languages*, however...

...some of them are more succinct

# Costs of the Optimal Simulations Between Automata

$$
\begin{tikzcd}
\text{1NFA} \arrow[r, "?"] \arrow[dr, "2^n"] & \text{2DFA} \arrow[d, "O(2^{n \log n})"] & \text{2NFA} \arrow[l, "?"'] \arrow[dl, "O(2^{n^2})"'] \\
& \text{1DFA} &
\end{tikzcd}
$$

[Rabin&Scott '59, Shepardson '59, Meyer&Fischer '71, ...]

### Question

*How much the possibility of moving the input head forth and back is useful to eliminate the nondeterminism?*

# Costs of the Optimal Simulations Between Automata

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& \text{1DFA} &
\end{tikzcd}$$

**Problem ([Sakoda&Sipser '78])**

*Do there exist polynomial simulations of*

- *1NFAs by 2DFAs*
- *2NFAs by 2DFAs ?*

**Conjecture**

*These simulations are not polynomial*

# Sakoda&Sipser Question: Upper and Lower Bounds

- Exponential upper bounds
  deriving from the simulations by 1DFAs

- Polynomial lower bounds
  for the cost $c(n)$ of simulation of 1NFAs by 2DFAs:
  - $c(n) \in \Omega(\frac{n^2}{\log n})$ [Berman&Lingas '77]
  - $c(n) \in \Omega(n^2)$ [Chrobak '86]

# Sakoda and Sipser Question

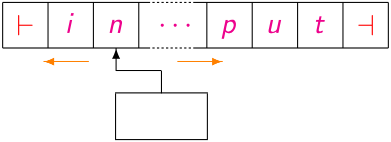
- Very difficult in its general form
- Not very encouraging obtained results:

  Lower and upper bounds too far
  (Polynomial vs exponential)

- Hence:

  Try to attack restricted versions of the problem!

# Two-Way Automata: Few Technical Details




- Input surrounded by the endmarkers $\vdash$ and $\dashv$
- $w \in \Sigma^*$ is accepted iff there is a computation
  - with input tape $\vdash w \dashv$
  - starting at the left endmarker $\vdash$ in the initial state
  - reaching a final state (on the left endmarker)

# 2NFAs vs 2DFAs: Restricted Versions

Previous works:

(i) Restrictions on the *simulating* machines (i.e., resulting 2DFAs)

- sweeping automata [Sipser '80]
- oblivious automata [Hromkovič&Schnitger '03]
- "few reversal" automata [Kapoutsis '11]

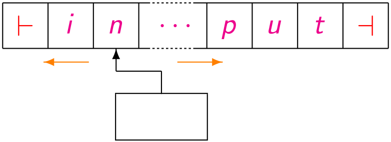
(ii) Restrictions on the *languages*

- unary regular languages [Geffert Mereghetti&Pighizzini '03]

In this work we use a different approach:

(iii) Restrictions on the *simulated* machines (i.e., given 2NFAs)

# Outer Nondeterministic Automata (ONFAs)




In the paper, we consider the following model:

### Definition

A two-way automaton is said to be *outer nondeterministic* iff nondeterministic choices are allowed *only* when the input head is scanning the endmarkers

# Unary 2NFAs vs ONFAs

## Normal Form for Unary 2NFAs [Geffert Mereghetti&Pighizzini '03]

- *Nondeterministic choices* only at the endmarkers
- *Head reversals* only at the endmarkers
- In each sweep the input length modulo one integer is counted

## Outer Nondeterministic Automata

- No restrictions on the *input alphabet*
- No restrictions on *head reversals*
- *Deterministic transitions* on "real" input symbols
- *Nondeterministic choices* only at the endmarkers

## Unary 2NFAs are a very restricted version of 2ONFAs!

- We extended to 2ONFAs previous results on unary 2NFAs

# Outer nondeterministic automata (ONFAs): tools

Main tool: procedure *reach*$(p, q)$

- Checks the existence of a computation segment
  - from the left endmarker in the state $p$
  - to the left endmarker in the state $q$
  - not visiting the left endmarker in between

*Accepting computation:*

sequence of states $q_0, q_1, \ldots, q_f$ visited at the left endmarker:

- $q_0$ initial state
- for $i = 1, \ldots, f$ $\mathit{reach}(q_{i-1}, q_i) = \mathit{true}$
- $q_f$ final state

# Outer nondeterministic automata (ONFAs): tools

- How to deal with loops?
- Two kinds of loops:
  - loops visiting the endmarkers
  - loops inside the "real" input

# Loops visiting the endmarkers

- Loops involving endmarkers can contain nondeterministic choices
- If a computation visits the left endmarker twice in the same state $q$ then there is a shorter "equivalent" computation
- We can consider only computations visiting the left endmarker $\leq \#Q$ times

# Loops inside the "real" input

Procedure *reach*$(p, q)$:

- "Backward search" from $q$ to $p$
- In this way loops are avoided
- Finite control with a *linear number of states*

The technique:

- Introduced by Sipser for the complementation of space bounded Turing machines [Sipser '80]
- Modified for the complementation of 2DFAs [Geffert Mereghetti&Pighizzini '07]
- Extended in our paper to 2ONFAs

# Results

(i) Subexponential simulation of 2ONFAs by 2DFAs
Verify that $q_f$ is reachable from $q_0$
by visiting the left endmarker $\leq \#Q$ times
(divide-and-conquere algorithm)

(ii) Polynomial complementation of 2ONFAs
Inductive counting argument

(iii) Polynomial simulation of 2ONFAs by 2DFAs
*under the condition* $\mathsf{L} = \mathsf{NL}$
Reduction to graph accessibility problem

(iv) Polynomial simulation of 2ONFAs by unambiguous 2ONFAs
Reduction to graph accessibility problem
combined with $\mathsf{NL}/\mathit{poly} \subseteq \mathsf{UL}$ [Reinhardt&Allender '00]

# Results: Alternating Case (2ONFAs)

At the endmarkers, universal and existential states are allowed

(v) Polynomial simulation of 2OAFAs by 2DFAs under $L = P$

(vi) Polynomial simulation of 2OAFAs by 2NFAs under $NL = P$

For both:
Reduction to the Alternating Graph Accessibility Problem

# Final Remarks

- We extended several results from the unary to the general case for 2ONFAs
- In the unary case, restricting the nondeterminism to the endmarkers does not significantly change the size of 2NFAs (normal form)
- In the general case, is there some "simple way" to restrict the nondeterminism?
- Does it is possible to extend our results to some wider class of 2NFAs?
- Interesting connections with complexity theory:
  - Results connected with classical complexity questions
  - Proof techniques derived from space complexity