Converting Self-Verifying Automata into Deterministic Automata

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Self-verifying machines

Standard machines (e.g. finite automata, pushdown automata, Turing machines) with nondeterministic transitions

The state set is partitioned in three groups:

- *accepting* states ("yes")
- *rejecting* states ("no")
- *neutral* states ("I do not know")

For each input word x the following conditions must be satisfyied:

- At least one computation on input x ends either in an accepting or in a rejecting state
- If a computation on x ends in an accepting state then there are no computations on x ending in rejecting states

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Some references:

- Ďuriš, Hromkovič, Rolim, and Schnitger (STACS 1997)
  Definition of the model in connection with the study of Las Vegas automata.
- Hromkovič and Schnitger (Information and Comp. 2001)
  Hromkovič and Schnitger (SIAM J. Comp. 2003)
  Further investigations in connection with Las Vegas computations and also *per se*.
- Assent and Seibert (RAIRO-ITA 2007)
  Simulation of self-verifying automata by deterministic automata.

Basic properties

- Trivial complementation
- Given nondeterministic machines M' and M'' for L and L^c, we can build a self-verifying machine M for L as the "union" of M' and M'', with a new initial state:

- Given a self-verifying machine for a language L we can easily obtain nondeterministic machines for L and for L^c

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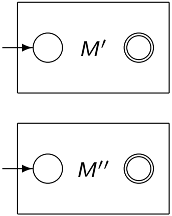

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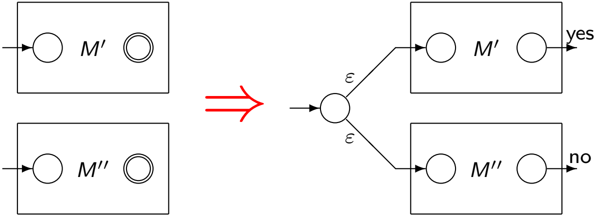

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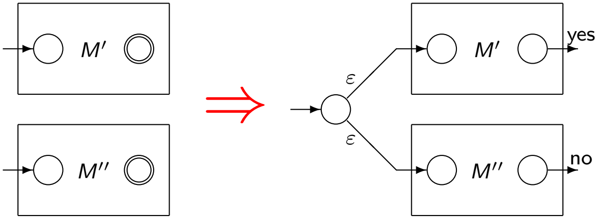

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Self-verifying automata (svfa): definition

$A = (Q, \Sigma, \delta, q_0, F^a, F^r)$ where:

- Q is the finite set of states
- Σ is the input alphabet
- $q_0 \in Q$ is the initial state
- $\delta : Q \times \Sigma \to 2^Q$ is the transition function
- $F^a \subseteq Q$ is the *set of accepting states*
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  namely, for each string there exists at least one accepting computation or one rejecting computation
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Languages of svfa's

We associate with an svfa A the following languages:

- The set of strings *accepted* by A:

$$L^a(A) = \{w \in \Sigma^* \mid \delta(q_0, w) \cap F^a \neq \emptyset\}$$

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What is the class of languages accepted by svfa's?

The answer to this question is easy:

- Each svfa is a nondeterministic automaton
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Hence:

Svfa's characterize the class of *regular languages*

Thus, each svfa can be converted into an equivalent dfa.

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How much it costs, in terms of states, the conversion of an n-state svfa into an equivalent dfa?

- Classical subset construction: upper bound 2^n
- It is possible to do better: Assent and Seibert (2007) reduced the upper bound to $O\left(\frac{2^n}{\sqrt{n}}\right)$, leaving open the optimality

In this work we further investigate this problem:

- We reduce the upper bound to a function $g(n)$ which grows like $3^{\frac{n}{3}}$
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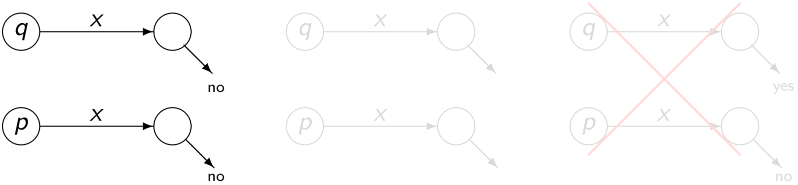

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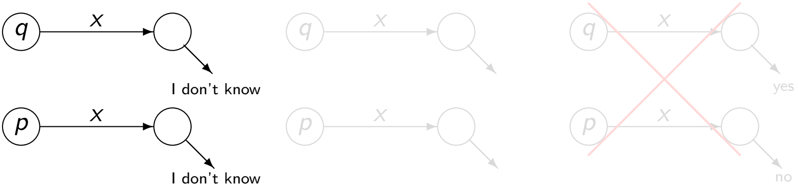

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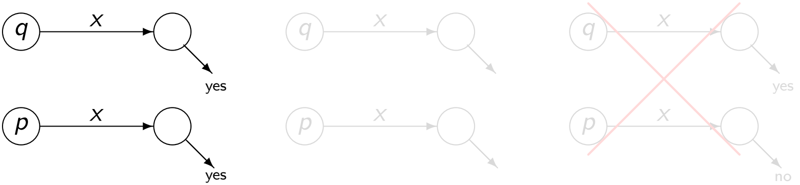

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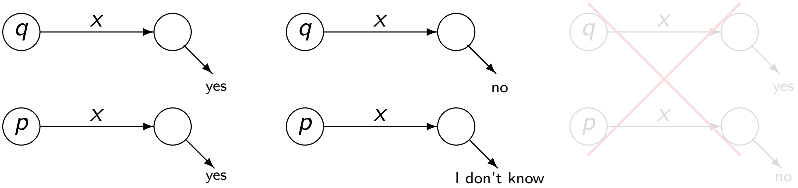

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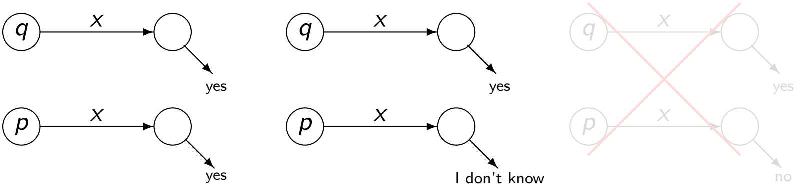

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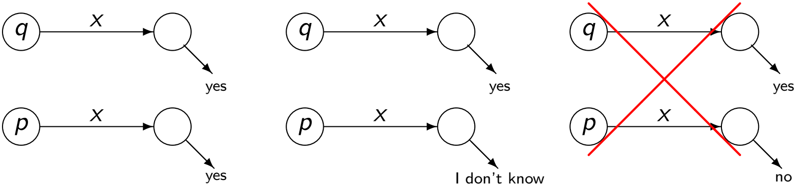

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Properties of the subset automaton

Let α be a state of the subset automaton A_{sub}. Then:

Each two states $q, p \in \alpha$ are compatible

Proof

If $q, p \in \alpha$ are not compatible then:

Given a string y s.t. α is reached on y, the original svfa on yx should give contradictory answers!

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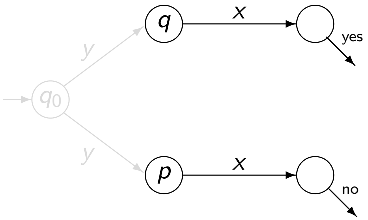

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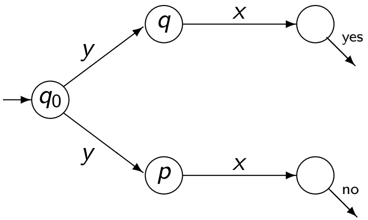

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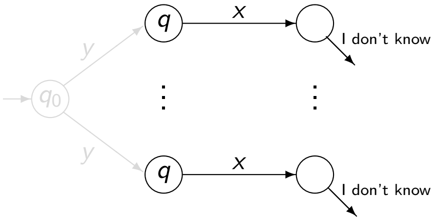

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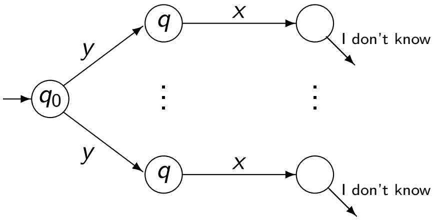

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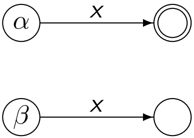

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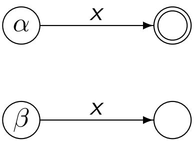
Let $\alpha, \beta \subseteq Q$ two states of A_{sub}

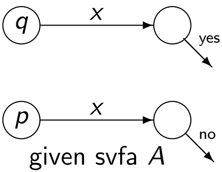
If $\alpha \cup \beta$ is a clique of G then α and β are equivalent

Proof
By contradiction, let x be a string distinguishing α and β:



subset automaton A_{sub}

Then $\exists q \in \alpha$, $p \in \beta$ s.t.:



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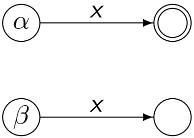

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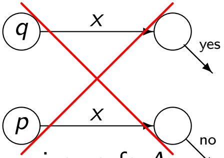

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- Each state of A_{sub} corresponds to a clique of the compatibility graph G
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Cliques in graphs

How many maximal cliques can a graph with n nodes have?

This question was answered by Moon and Moser (1965).

They proved the following *exact bound* $f(n)$ for the maximum number of maximal cliques in a graph with n nodes:

$$f(n) = \begin{cases} 3^{\lfloor \frac{n}{3} \rfloor} & \text{if } n \equiv 0 \pmod 3 \\ 4 \cdot 3^{\lfloor \frac{n}{3} \rfloor - 1} & \text{if } n \equiv 1 \pmod 3 \\ 2 \cdot 3^{\lfloor \frac{n}{3} \rfloor} & \text{if } n \equiv 2 \pmod 3 \end{cases}$$

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Conversion of svfa's into dfa's: upper bound

Using the result of Moon and Moser, we can prove that

Each n-state svfa's can be simulated by a dfa with at most $g(n) = 1 + f(n-1)$ states

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- We proved that A_{sub} can be reduced to a dfa with at most one state for each maximal clique of G
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Notice that $g(n) = O(3^{\frac{n}{3}})$

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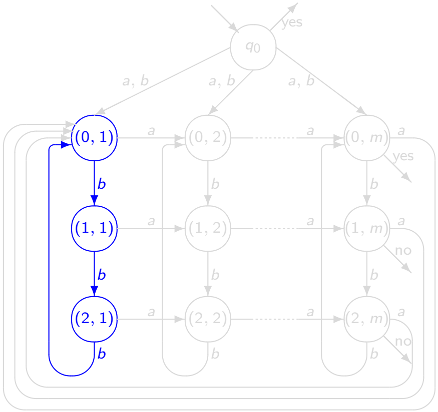

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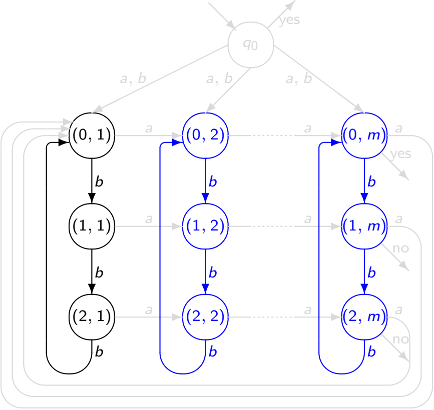

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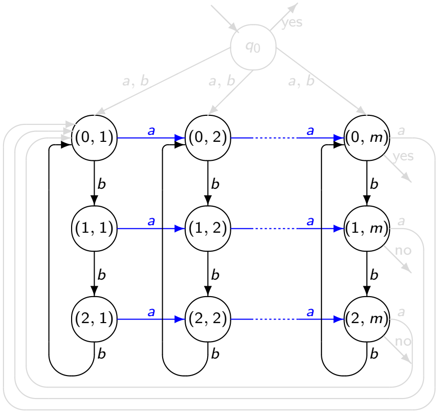

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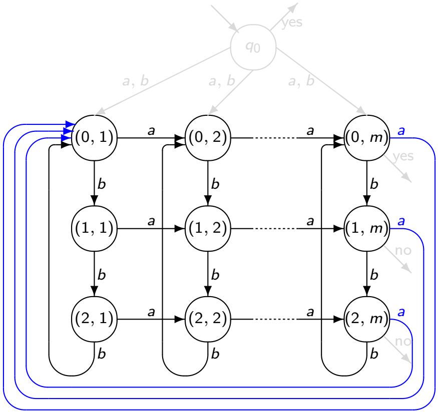

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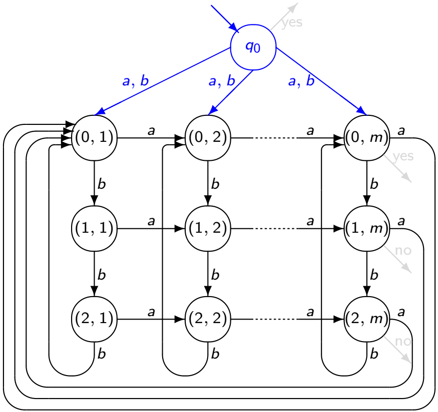

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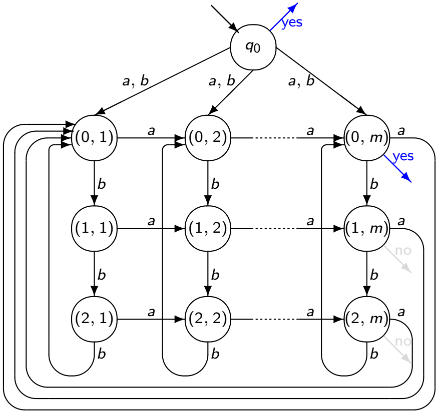

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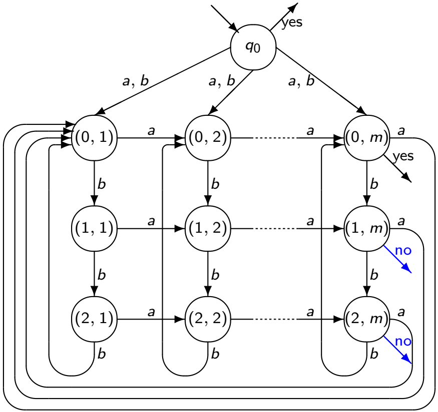
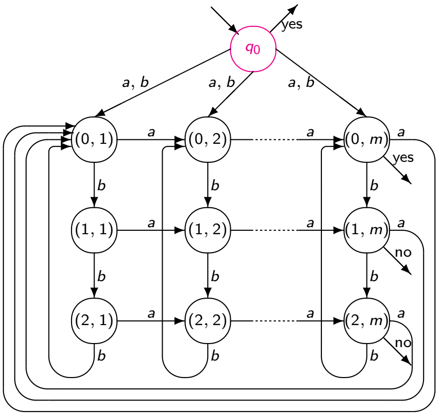


Properties of A_n



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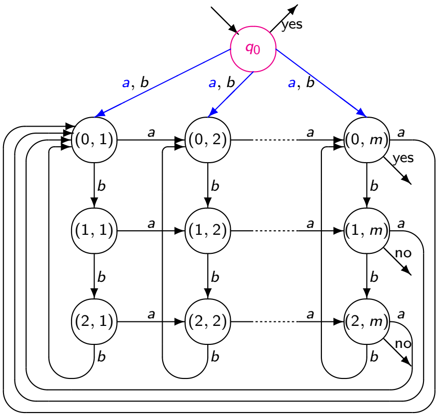
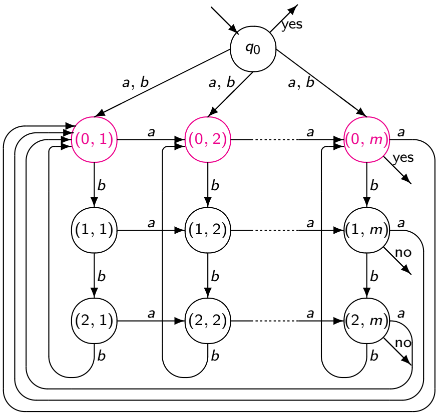


a


yes
q_0
a, b
a, b
a, b
(0, 1)
a
(0, 2)
a
(0, m)
a
yes
b
b
b
(1, 1)
a
(1, 2)
a
(1, m)
a
no
b
b
b
(2, 1)
a
(2, 2)
a
(2, m)
a
no
b
b
b

a

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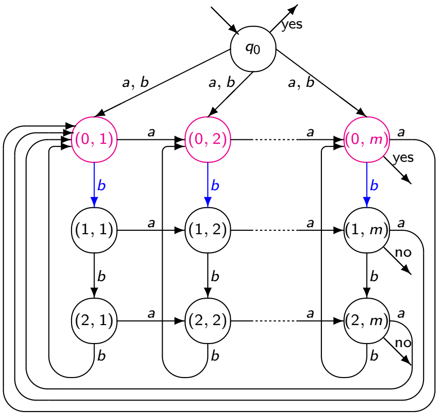

ab

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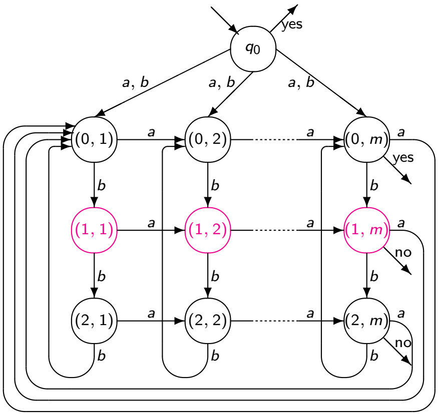
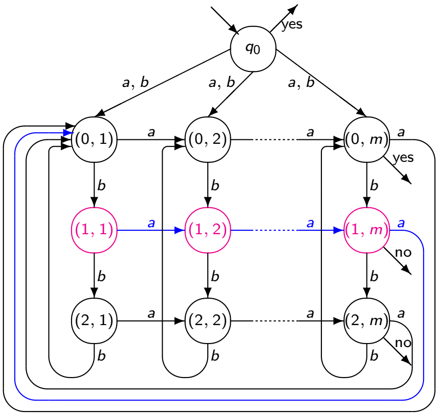


ab



q0
yes
a, b
a, b
a, b
(0, 1)
a
(0, 2)
a
(0, m)
a
yes
b
b
b
(1, 1)
a
(1, 2)
a
(1, m)
a
no
b
b
b
(2, 1)
a
(2, 2)
a
(2, m)
a
no
b
b
b

aba

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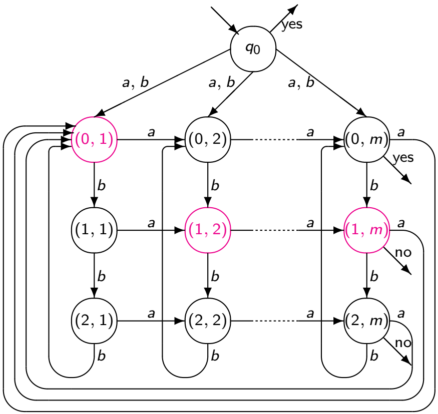

aba

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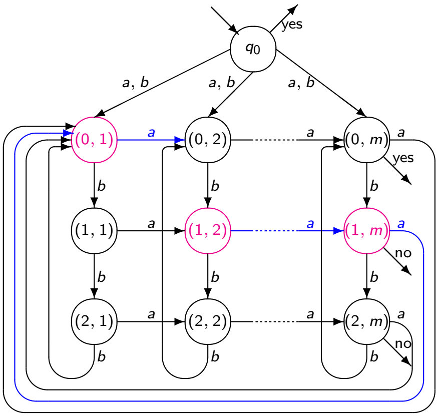

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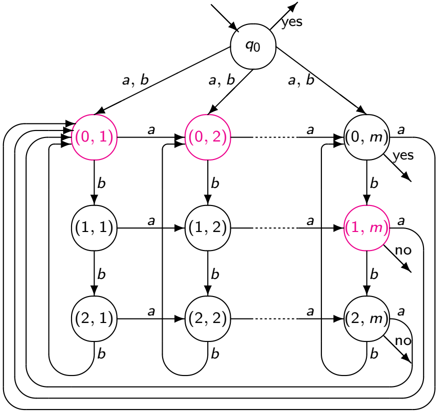

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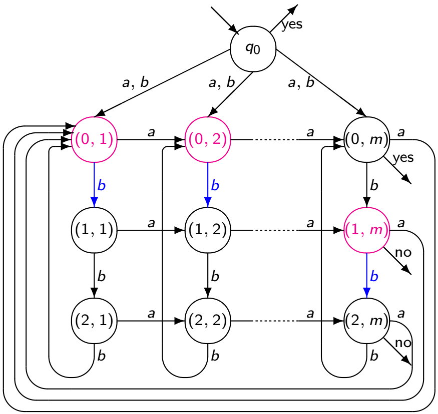

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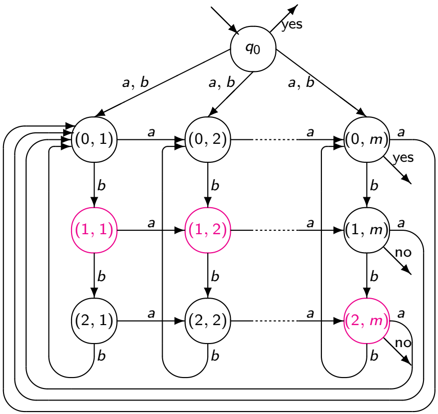

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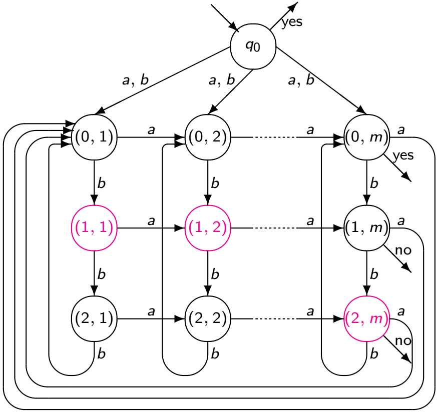
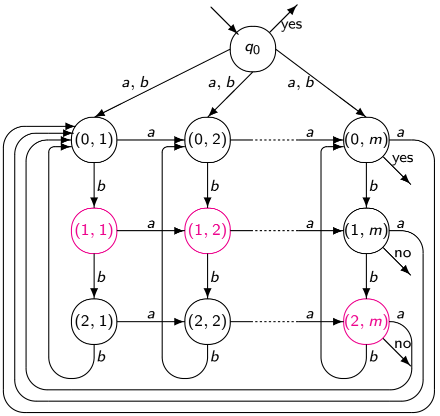


The reachable states of the subset automaton A_{sub} are:

- $\{q_0\}$
- the 3^m subsets obtained by taking one state from each column in the "grid part" (hence A_n is an svfa!)

Properties of A_n



- We can verify that each two states of A_{sub} are distinguishable

Summing up:

- The subset automaton A_{sub} has exactly $g(n) = 1 + 3^{\frac{n-1}{3}}$ states
- All these states are pairwise distinguishable
- Hence, it is the minimal dfa equivalent to A_n
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Hence:

the *exact cost* for the conversion of n-state svfa's into equivalent dfa's is:

$$g(n) = \begin{cases} 1 + 3^{\frac{n-1}{3}} & \text{if } n \equiv 1 \pmod 3 \text{ and } n \geqslant 4 \\ 1 + 4 \cdot 3^{\frac{n-2}{3}-1} & \text{if } n \equiv 2 \pmod 3 \text{ and } n \geqslant 5 \\ 1 + 2 \cdot 3^{\frac{n}{3}-1} & \text{if } n \equiv 0 \pmod 3 \text{ and } n \geqslant 3 \\ n & \text{if } n \leqslant 2 \end{cases}$$

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Svfa's with multiple initial states

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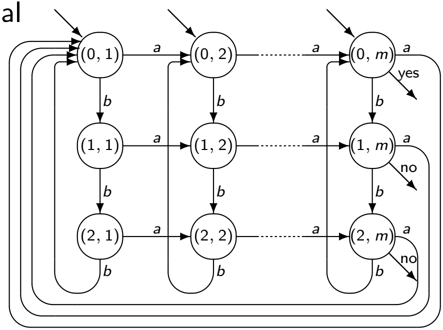

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- We proved the optimality using automata over a binary alphabet
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- In fact, if L is a unary language accepted by a n-state unary nfa such that the minimal dfa for L requires $F(n)$ states, then each nfa for L^c requires $F(n)$ states (Mera, Pighizzini, 2005)

The unary case

What about the optimality in the unary case?

- We proved the optimality using automata over a binary alphabet
- The cost of the conversion of unary nfa's into dfa's is $F(n) = \mathrm{e}^{O(\sqrt{n \log n})}$ (Chrobak, 1986)
- $F(n)$ grows more slowly than $g(n)$.
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Conclusion

- Each n-state svfa can be converted into an equivalent dfa with $g(n)$ states:
  - We found the value of $g(n)$, which grows like $3^{\frac{n}{3}}$
  - *The bound is exact:*
    for each integer n, there exists an svfa A_n with n states and an input alphabet of two letters such that the minimal equivalent dfa has $g(n)$ states.

- Each n-state svfa with multiple initial states can be converted into an equivalent dfa with $f(n) = g(n+1) - 1$ states.
  Also this bound is exact.

- In the unary case, a better upper bound is given by the function $F(n) = e^{O(\sqrt{n \log n})}$.
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